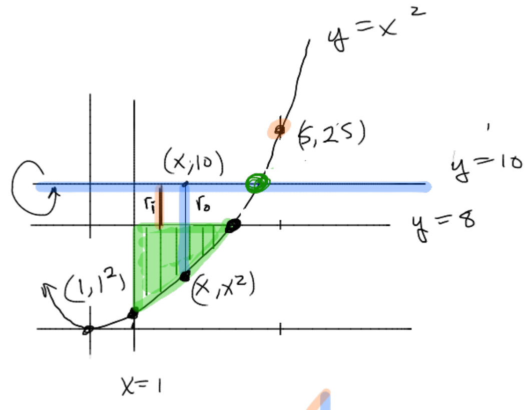
warm-up wk 12

Volume of solid: axis of revolution is $\underline{\text{not}}$ x-axis

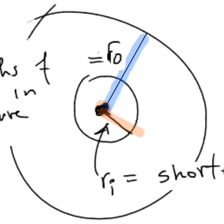
Region: $x = 1$, $y = x^2$, $y = 8$

① visual



Revolve around: $y = 10$

② slice ⊥ axis, since axis of rev. is $\underline{\underline{\text{not}}}$ attached to region ⇒ get washer as slice



r_i = shortest dist b/w axis and region $= 2$

Key: center of washer lies on axis of rev.

set $x^2 = 8$

$\boxed{2\sqrt{2}}$

③ Integrate along axis of rev. ($y = 10$ is horizontal ⇒ x changes ⇒ Int. w.r.t. x)

$$\int_{\min x}^{\max x} \pi\left(r_o^2 - r_i^2\right) dx = \int_1^{2\sqrt{2}} \pi\left[\left(10 - x^2\right)^2 - 2^2\right] dx$$

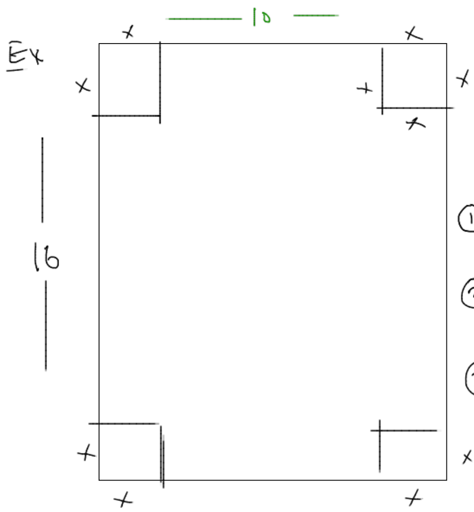
$$= \pi \int_1^{2\sqrt{2}} 100 - 20x^2 + x^4 - 4 \, dx = \pi\left(100x - \frac{20x^3}{3} + \frac{x^5}{5} - 4x\right)\Bigg|_1^{2\sqrt{2}} =$$

Other Applications

Differentiation: (Optimization)

eg, word problems involving the words 'most', 'least', 'greatest', 'largest', 'shortest', ..

Ex

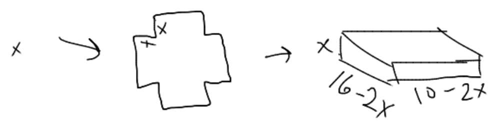


Start with a 10 * 16 inch sheet of cardboard. Cut out a square from each corner and fold up & glue the sides, to make an open box. The size of the box will depend on the size of square you remove. What is the largest volume (box) that can be made?

① Identify what's the goal: Largest Box ⟹ create function for Volume, maximize it (① take deriv. ② set = 0 ③ solve)

② create variables to represent what's changing
x = side length of square

③ relate variables to function:



$$V = (x)(16-2x)(10-2x)$$

④ Prepare take deriv.

$$V = x(160 \underbrace{- 32x - 20x} + 4x^2) = 4x^3 - 52x^2 + 160x$$

$$V'(x) = 12x^2 - 104x + 160 = 0$$

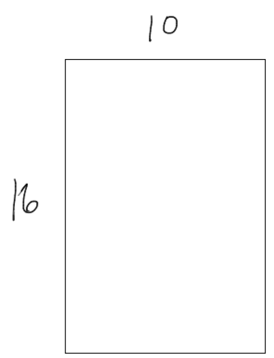
$$6x^2 - 52x + 80 = 0$$

$$V'(x) = 3x^2 - 26x + 40 = 0$$

$$QF \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{26 \pm \sqrt{26^2 - 4 \cdot 3 \cdot 40}}{6}$$

$$= \frac{26 \pm \sqrt{26^2 - 480}}{6} = \{2, 6.6\}$$

$x = 2$ gives largest vol



What is the volume of largest box?

V(2) = $2(16-8)(10-4)$

$= 2 \cdot 8 \cdot 6 = 96 \text{ in}^3$