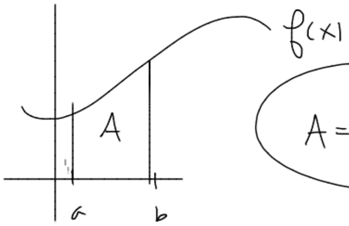
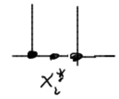
Mon Wk 12

Area under curve



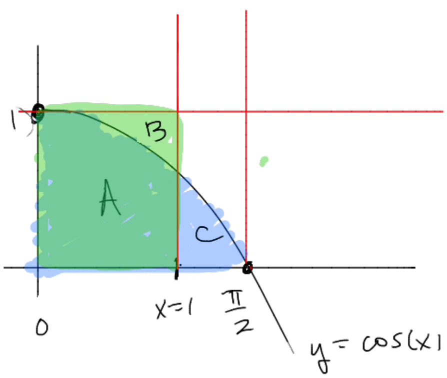
$$A = \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$



some x-value in ith sub-interval

definite integral

warm-up.



Find the area A below

$$\int_{\text{left}}^{\text{right}} \text{function} = \int_0^{\frac{\pi}{2}} \cos(x)\,dx = \sin(x)\Big|_0^{\pi/2}$$

$$= \sin\left(\tfrac{\pi}{2}\right) - \sin(0) = 1$$

So Area B = Area C

PROPERTIES of the definite integral

① Fundamental theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

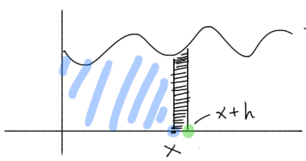
w/ F = an anti-derivative of $f(x)$

② FTC

area so far function

$$A(x) = \int_0^x f(t)\,dt$$

= area under graph of $f(t)$ b/w 0 and x



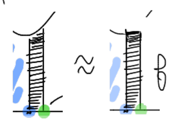
and

$$\boxed{A'(x) = f(x)}$$

Proof:

$A(x+h)$ = blue portion + black portion

If h is small



$\approx f(x+h)\cdot h$

$$A(x+h) - A(x) \approx f(x+h)\cdot h$$

now divide by h $\quad \dfrac{A(x+h) - A(x)}{h} = f(x+h)$ $\quad$ and take $\lim_{h\to 0}$

$$A'(x) = \lim_{h\to 0} \frac{A(x+h) - A(x)}{h} = \lim_{h\to 0} f(x+h) = f(x)$$

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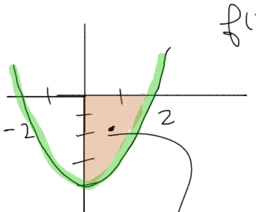
③ $\int_a^b f(x)\,dx$ computes signed area

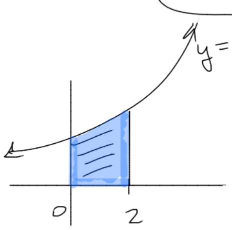
$f(x) = x^2 - 4$



$$\int_0^2 x^2 - 4\,dx = \frac{x^3}{3} - 4x\Big|_0^2 = \frac{8}{3} - 8 - \left[\frac{0^3}{3} - 4\cdot 0\right] = -5.33$$

below x-axis

Here, notice $x = 2 > 0$ (bigger # on bottom)



$$\int_2^0 e^x\,dx = e^x\Big|_2^0 = e^0 - e^2 = 1 - e^2 \approx 1 - 8.1 = -7.1$$

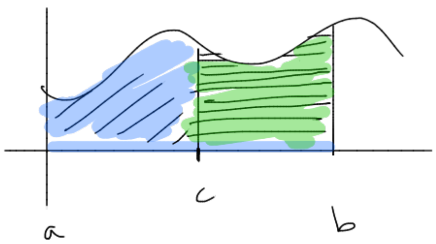
here integrate from R to L

④ It doesn't matter which anti-deriv you use:

$$\int_0^4 x^2\,dx = \frac{x^3}{3} + C\Big|_0^4 = \frac{4^3}{3} + C - \left(\frac{0^3}{3} + C\right) = \frac{4^3}{3} - 0 + \underbrace{C - C}_{=0}$$

More properties

⑤ Additive Property



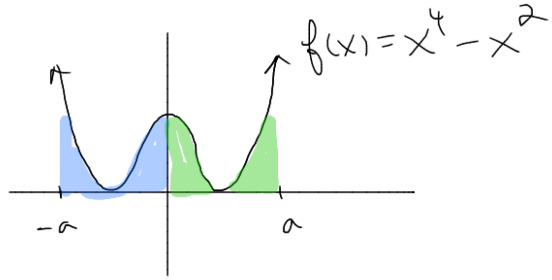
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

⑥ Even Property

$f(-x) = f(x)$

$\Downarrow$

symmetric about y-axis



If $f(x)$ is even

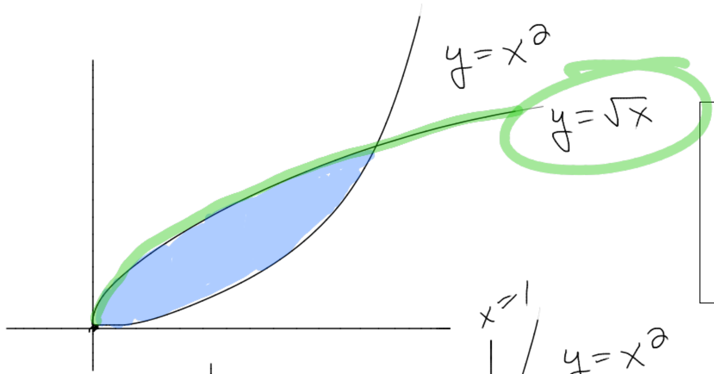
$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

Ex Compute area under $f(x) = x^4 - x^2$ from -3 to 3.

option $\displaystyle\int_{-3}^{3} x^4 - x^2\,dx = \frac{x^5}{5} - \frac{x^3}{3}\Big|_{-3}^{3} = \frac{3^5}{5} - \frac{3^3}{3} - \left[\frac{(-3)^5}{5} - \frac{(-3)^3}{3}\right] = \underbrace{\frac{3^5}{5} - \frac{3^3}{3}}_{\text{same}} + \underbrace{\frac{3^5}{5} - \frac{3^3}{5}}_{\text{same}}$

$$= 2\left(\frac{3^5}{5} - \frac{3^3}{3}\right)$$

Area between curves



Compute area b/w the two graphs

$$\boxed{\int_0^1 x^2-\sqrt{x}\,dx = \int \text{Top}-\text{bottom} \qquad = \frac{x^3}{3}-\frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{1}{3}-\frac{2}{3} = -\frac{1}{3}}$$

we got top/bottom swapped

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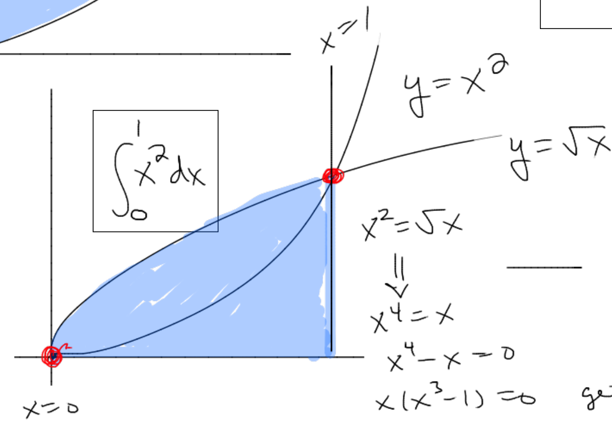

$x^2=\sqrt{x}$

$\Downarrow$

$x^4=x$

$x^4-x=0$

$x(x^3-1)=0$ get $x=0$ $x=1$

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