Wk 12 Tue

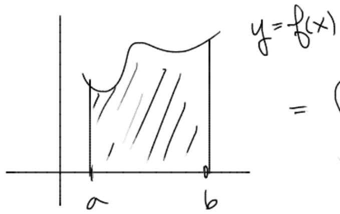
Remaining

wk 12 area b/w curves (apps of integration)

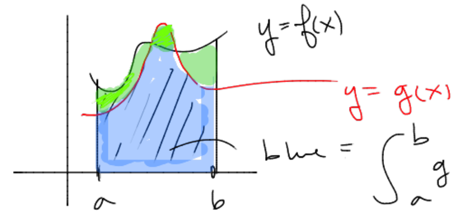
wk 13 volumes

wk 14 exam 4 / review

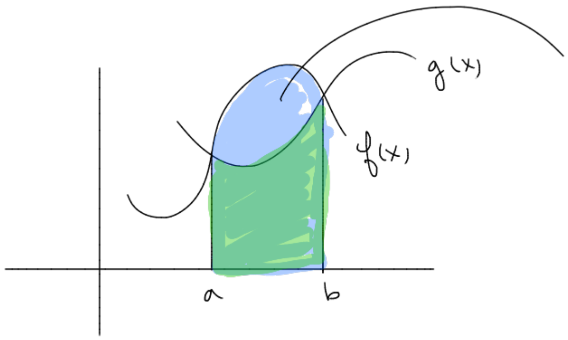
Area:



$$= \int_a^b f(x)\,dx$$



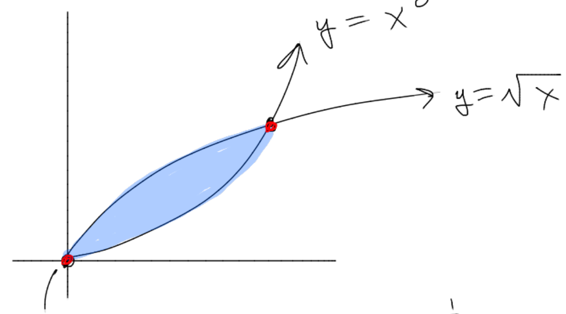
$$\text{blue} = \int_a^b g(x)\,dx$$



$$\text{blue} = \int_a^b \text{top} - \text{bottom}\,dx$$

$$= \boxed{\int_a^b f(x) - g(x)\,dx}$$

Ex.



Q: Compute area of shaded region.

① Bounds of integration
(min & max x-values when int. wrt. x)

set two functions equal:

$$x^2 = \sqrt{x}$$

↓ square

$$x^4 = x$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$\boxed{x = 0}$ $a = 0$

$x^3 - 1 = 0$

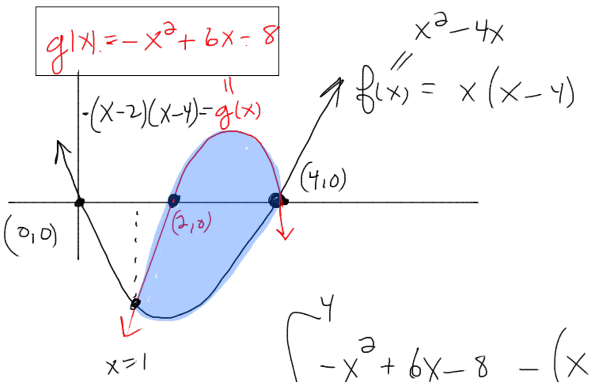
$\boxed{x = 1}$ $b = 1$

$$\int_a^b \text{top} - \text{bottom}\, dx = \int_0^1 \sqrt{x} - x^2\, dx$$

$\sqrt{x} = x^{\frac{1}{2}}$

$$= \frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big|_0^1 = \frac{2}{3}(1)^{3/2} - \frac{1^3}{3} - \left[\frac{2}{3}0^{3/2} - \frac{0^3}{3}\right] = \frac{1}{3} \text{ sq. units}$$

Q: what is the area of shaded region?



① Bounds:

set functions = & solve

$$-x^2+6x-8 = x^2-4x$$

$$0 = 2x^2-10x+8$$

$$0 = x^2-5x+4$$

$$= (x-4)(x-1)$$

$x = 4 \rightarrow b$

$x = 1 \rightarrow a$

$$\int_1^4 -x^2+6x-8-(x^2-4x)\,dx$$

$$= \int_1^4 -2x^2+10x-8\,dx = -\frac{2x^3}{3}+\frac{10x^2}{2}-8x\Big|_1^4$$

$$= -\frac{2(4)^3}{3}+\frac{10(4)^2}{2}-8(4)-\left[\underbrace{-\frac{2(1)^3}{3}+\frac{10(1)^2}{2}-8(1)}_{-3}\right] = -\frac{2(\overset{63}{64})}{3}+80-32-\left[-3.\bar{6}\right]$$

$$\approx \underbrace{-42+80-32}_{8} \approx 11.\bar{6}$$

$y = 3\sin x$

$y = 2\cos x$

Find area shown!

$a = -\frac{\pi}{2}$

$b = \frac{\pi}{2}$

Bounds?

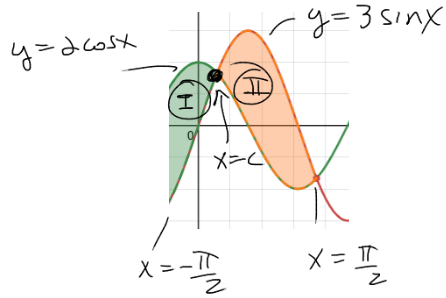
Set fcns =

$2\cos x = 3\sin x$

$2 = 3\,\frac{\sin x}{\cos x}$

$\frac{2}{3} = \frac{\sin x}{\cos x} = \tan x$

$\underbrace{\tan^{-1}\left(\frac{2}{3}\right)}_{\approx .58} = \underline{\tan^{-1}(\tan x)} = x$



I $\displaystyle\int_{-\frac{\pi}{2}}^{c = .58} 2\cos x - 3\sin x \, dx$

Focus: start

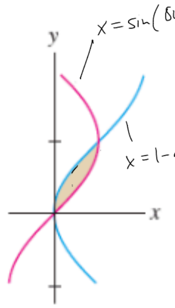
II $\displaystyle\int_{c = .58}^{\pi/2} 3\sin x - 2\cos x \, dx$

ans. I + II

$= .60$

Question 2 of 8

Find the area between the graphs $x = \sin(8y)$ and $x = 1 - \cos(8y)$ over the interval $0 \leq y \leq \frac{\pi}{16}$ in the figure.



$x = \sin(8y)$

$x = 1 - \cos(8y)$

$$\int_{\min y}^{\max y} \text{Right Most} - \text{Left Most} \; dy$$

$$\int_0^{\frac{\pi}{16}} \sin(8y) - (1 - \cos(8y)) \, dy$$

$u = 8y$ | if $y = \pi/16 \Rightarrow u = \frac{8\pi}{16} = \frac{\pi}{2}$

$du = 8\,dy$ | $y = 0 \Rightarrow u = 8 \cdot y = 0$

$\frac{1}{8} du = dy$

Changed!! →
$$\int_0^{\pi/2} \sin(u)\frac{1}{8}\,du - 1 + \cos(u)\frac{1}{8}\,dy = \frac{1}{8}\int_0^{\pi/2} \sin(u) - 1 + \cos(u)\,du$$