Wk 12 Tue

Remaining

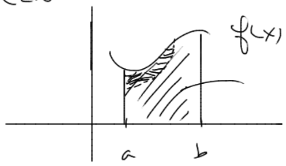
wk 12 area b/w curves (apps of integration)

wk 13 volumes

wk 14 exam 4 / review

Area b/w curves

Recall

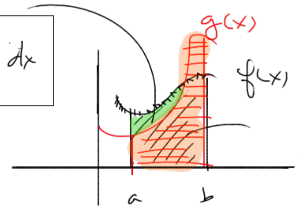


$\int_a^b f(x)$

$$\int_a^b f(x) - g(x)\,dx$$

green

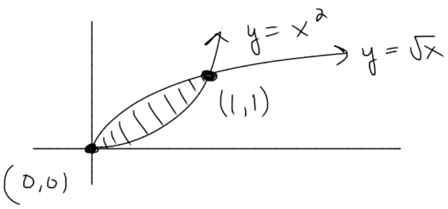
$\int$ Top - bottom



Orange $= \int_a^b g(x)\,dx$

$\int_a^b f(x)$

Ex



Compute shaded area:

1. bounds of integration (where curves intersect / domain of region) (set functions equal)
   (if int. wrt. x, use x-values of intersection points)

set $\sqrt{x} = x^2$ solve:

$\downarrow$ square

$x = x^4$

$0 = x^4 - x = x(x^3 - 1)$

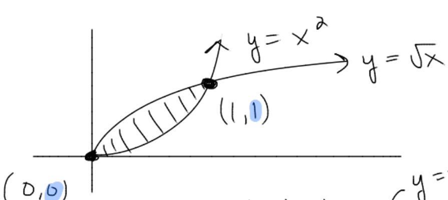
$x = 0,\ x = 1 \Rightarrow \boxed{a = 0 \quad b = 1}$

2. $\int_a^b$ top − bottom

$$\int_0^1 \sqrt{x} - x^2\,dx$$

$$= \frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big|_0^1 = \frac{2}{3} - \frac{1}{3} - [0 - 0] = \boxed{\frac{1}{3} \text{ sq unit}}$$

Note: Sometimes it's more convenient to integrate wrt. y.



wrt y

1. get functions in terms of y.

$y = x^2$ isolate x: $\sqrt{y} = x$

$y = \sqrt{x}$ $\Rightarrow$ $y^2 = x$

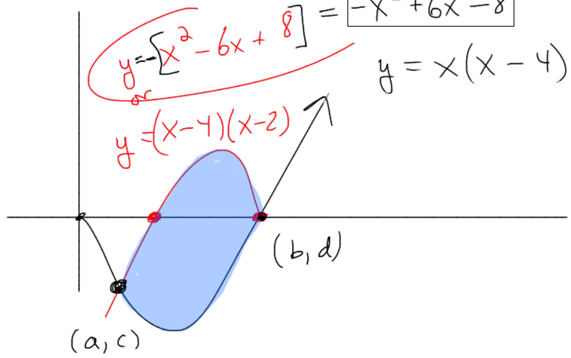
$y = x^2\ (x = \sqrt{y})$ $\quad$ $y = \sqrt{x}\ (x = y^2)$

$$\int_{\min y}^{\max y} \text{Right most} - \text{Left most}\ dy = \int_0^1 \sqrt{y} - y^2\,dy = \frac{1}{3}$$

Ex It doesn't matter if one is below x-axis.

$= \boxed{-x^2+6x-8}$

$y = x(x-4) = x^2-4x$



Find area b/w these curves

Int. wrt x! from a to b.

1 Bounds: "b": where $x^2-4x = -x^2+6x-8$ (set functions equal)

$2x^2-10x+8=0$ so $b=4$

$x^2-5x+4=0$ $a=1$

$(x-4)(x-1)=0 \Rightarrow x=4,\ x=1$

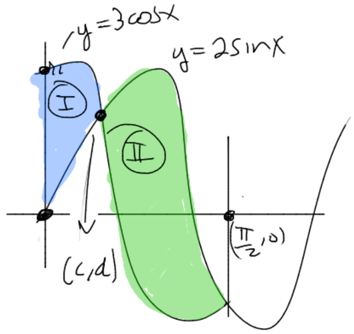
2 $\int_1^4 -x^2+6x-8-(x^2-4x)\,dx = \int_1^4 -2x^2+10x-8\,dx$

$= -\frac{2x^3}{3}+\frac{10x^2}{2}-8x\Big|_1^4 = -\frac{2}{3}(4)^3+5(4)^2-8(4)-\left[-\frac{2}{3}(1)^3+\underbrace{5(1)^2-8(1)}_{-3}\right]$

$= -\frac{2}{3}64 + \underbrace{\underbrace{80-32}_{48}+3.\bar{6}}_{51.\bar{6}}$ $\simeq 51.\bar{6}-42$

$-42 \simeq -\frac{2}{3}63$ $\approx 8.\bar{3}$

Care w/ switching



$f(x) = 2\sin(x)$

$g(x) = 3\cos(x)$

$b = \frac{\pi}{2}$

$a = 0$

Find area between the curves

Bounds: set equations =

$3\cos x = 2\sin x$

$\div \cos x$

$3 = 2\frac{\sin x}{\cos x}$

$\frac{3}{2} = \frac{\sin x}{\cos x} = \tan x$

$x = \tan^{-1}\left(\frac{3}{2}\right)$

$\approx .98$

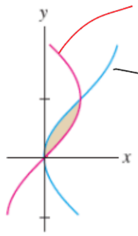
$$\text{I} = \int_0^c 3\cos x - 2\sin x \, dx$$

$$= 3\sin x + 2\cos x \Big|_0^{.98}$$

$$\text{II} = \int_c^{\frac{\pi}{2}} 2\sin x - 3\cos x \, dx = -2\cos x + 3\sin x \Big|_{.98}^{\pi/2}$$

Question 2 of 8

Find the area between the graphs $x = \sin(8y)$ and $x = 1 - \cos(8y)$ over the interval $0 \le y \le \frac{\pi}{16}$ in the figure.



$x = \sin(8y)$

$x = 1 - \cos(8y)$

$$\int_{\min y}^{\max y} \text{Right} - \text{left} \; dy = \int_0^{\pi/16} \sin(8y) - (1 - \cos(8y))\,dy$$

$u = 8y$

$du = 8\,dy$

$\frac{1}{8} du = dy$

$y = 0 \Rightarrow u = 8 \cdot 0$

$y = \pi/16 \Rightarrow u = 8 \cdot \frac{\pi}{16}$

$u = \frac{\pi}{2}$

$$\int_0^{\pi/2} \sin(u) \frac{1}{8}\,du = -\frac{1}{8}\cos(u) \Big|_0^{\pi/2}$$

$$= -\frac{1}{8}\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= -\frac{1}{8}(0 - 1)$$

$$= \frac{1}{8}$$