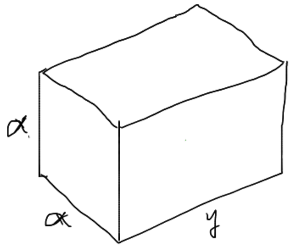
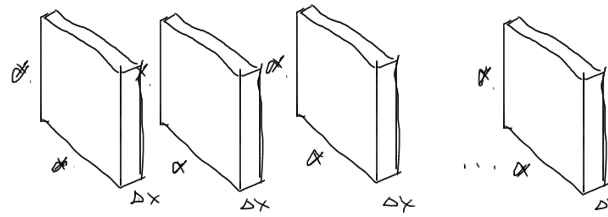
Thurs wk 12



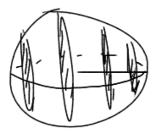
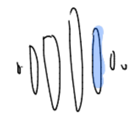
Vol. of Stick of butter: $= \alpha \cdot \alpha \cdot y = \alpha^2 y$



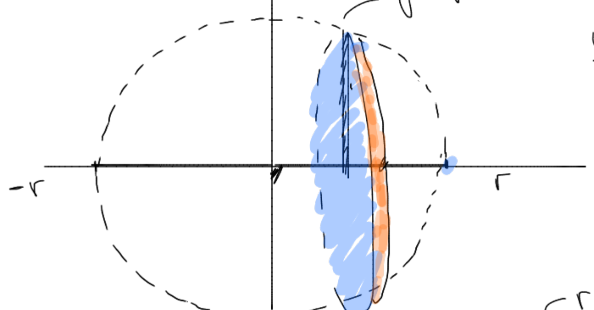
as $\Delta x \to 0$ get thinner

$$= \int_0^y \text{slice area}\, dx = \int_0^y \alpha^2\, dx = \alpha^2 \int_0^y 1\, dx = \alpha^2 \cdot x \Big|_0^y = \alpha^2\left[(y) - 0\right] = \alpha^2 y$$

Another Classic Formula:

Vol of Sphere:  $V = \frac{4}{3}\pi r^3$ = 

To derive via calc: look @ half-slice



Vol of Slice. $\pi R^2 = \pi(r^2 - x^2)$

$$\int_{-r}^{r} \text{Area}\, dx = \int_{-r}^{r} \pi(r^2 - x^2)\, dx$$

even function (deg = 2)

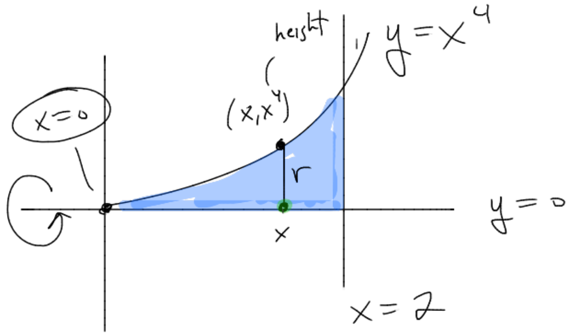
$$= 2\int_0^r \pi(r^2 - x^2)\, dx = 2\pi\left[r^2 x - \frac{x^3}{3}\right]_0^r = 2\pi\left[r^2 r - \frac{r^3}{3}\right]$$

$$2\pi\left[r^3 - \frac{r^3}{3}\right]$$

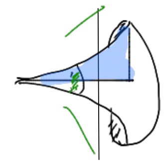
$$= 2\pi\left[\frac{3r^3}{3} - \frac{r^3}{3}\right]$$

$$= 2\pi\left[\frac{2r^3}{3}\right] = \frac{4\pi r^3}{3}$$

Ex Find the volume of the solid obtained by rotating the shaded area about the x-axis



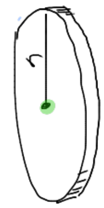
① get visual :



② slice ⊥ to axis of revolution

x-axis is horizontal ⇒ slice vertically

key: center of disc is <u>ON</u> the axis



r = dist b/w axis and curve

$r = x^4$

③ Vol of Slice = Area of disc × thickness

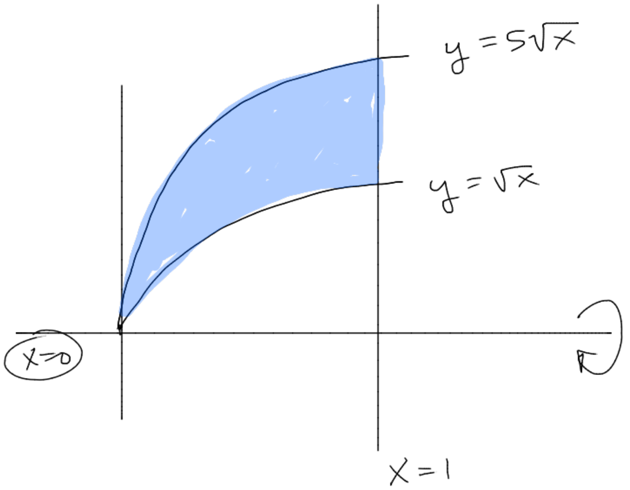
$= \pi r^2 \cdot \Delta x$

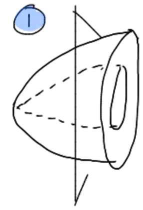
$= \pi (x^4)^2 \Delta x = \pi x^8 \Delta x$

④ Integrate along axis of rev.

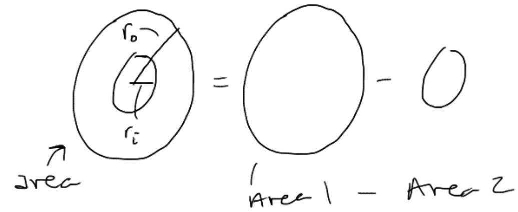
$$\int_{\min x}^{\max x} \text{area}\, dx$$

$$= \int_0^2 \pi x^8\, dx = \frac{\pi x^9}{9}\Big|_0^2 = \frac{512\pi}{9} \text{ cubic units}$$



① 

② slice ⊥ axis of rev,



key: center of disc is on axis

③ $A_1 = \pi r_o^2 = \pi(5\sqrt{x})^2$

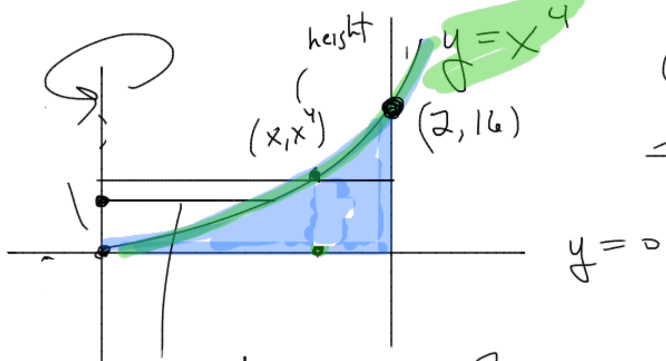
$A_2 = \pi r_i^2 = \pi(\sqrt{x})^2$

$A = A_1 - A_2 = \pi(r_o^2 - r_i^2)$

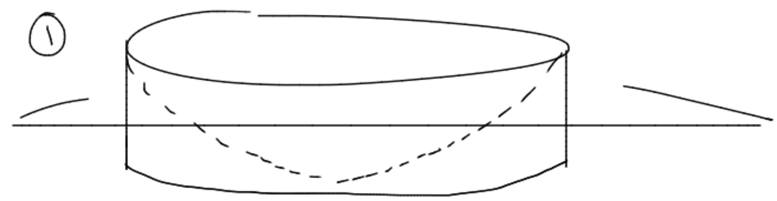
$A = \pi(25x - x) = 24\pi x$

④ $\int_0^1 24\pi x \, dx = \left. \frac{24\pi x^2}{2} \right|_0^1 = \frac{24\pi}{2} = 12\pi$ cubic units

Revolve about y axis

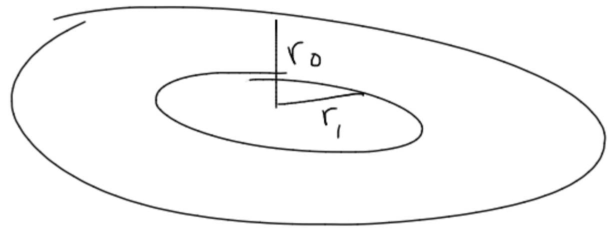


length $=$ x-coord, get x-coord via y.

① 

② slice $\perp$ axis

$\Rightarrow$ horizontal



$r_1 = x$ w/ $y = x^4$

$r_1 = \sqrt[4]{y}$ $\quad \sqrt[4]{y} = x$

r_0, r_1 are functions of y.

center of annulus is on axis

$r_0 = 2$

$$\int_0^{\max y = 16} \pi (r_0^2 - r_1^2)\, dy = \int_0^{16} \pi \left(2^2 - (\sqrt[4]{y})^2\right) dy$$