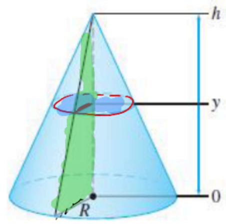
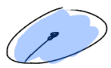
wk 12 thur

6-2: Volume!

6.2.2

Let V be the volume of a right circular cone of height $h = 20$ whose base is a circle of radius $R = 4$. Compute Volume V.



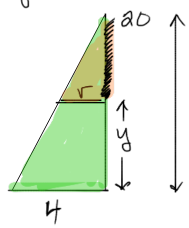


Strategy: $V = \int$ area of slice

① Find the area of slice; slice = circle, $A = \pi r^2$

② Find a formula for r, radius of a slice (r = function of y) $A = \pi r^2$

③ Key to this is geometry.



$$\frac{\text{short leg}}{\text{long leg}} = \frac{4}{20} = \frac{r}{20-y} \Rightarrow r = \frac{4(20-y)}{20} = \boxed{\frac{1}{5}(20-y) = r}$$

④ $A = \pi r^2 = \pi\left(\frac{1}{5}(20-y)\right)^2 = \frac{\pi}{25}(20-y)^2$

⑤ $\int_0^{20} \frac{\pi}{25}(20-y)^2\,dy$

⑥ $u = 20 - y$

$du = -dy$

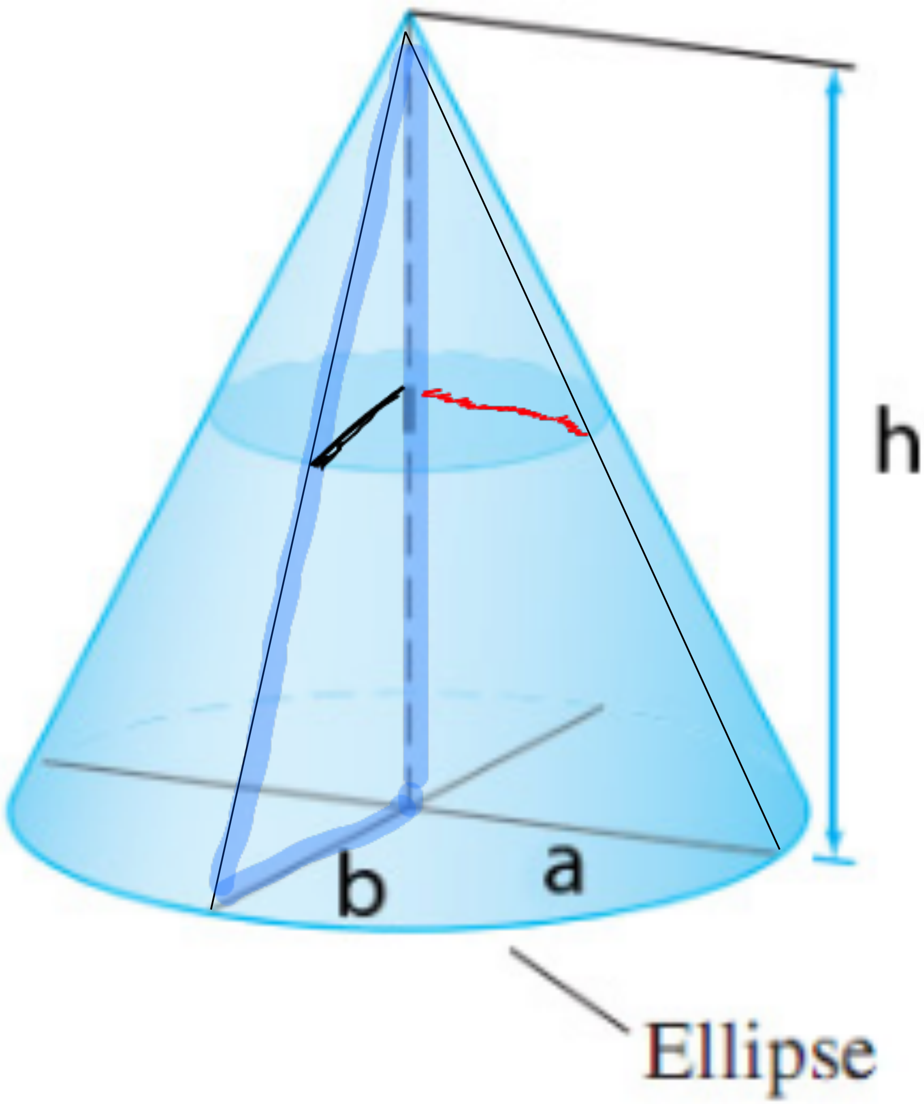
$-du = dy$

when $y = 0$, $u = 20 - 0 = 20 = u$

$y = 20$, $u = 20 - 20 = 0 = u$

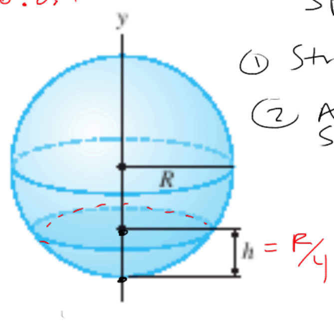
$$= \frac{\pi}{25}\int_{20}^{0} u^2(-du) = -\frac{\pi}{25}\int_{20}^{0} u^2\,du = \frac{-\pi}{25}\cdot\frac{u^3}{3}\Big|_{20}^{0}$$

$$= \frac{-\pi}{25}\left[\frac{0^3}{3} - \frac{20^3}{3}\right] = \frac{-\pi}{25}\left(-\frac{20^3}{3}\right) = \frac{\pi}{5\cdot5}\cdot\frac{\overset{5\cdot4}{20}\cdot\overset{5\cdot4}{20}\cdot20}{3} = \boxed{\frac{320\pi}{3}}$$



FIGURE

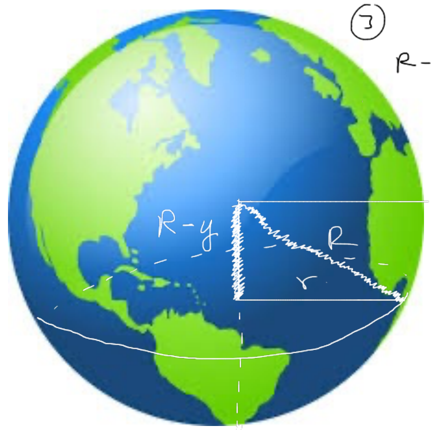
6.2.4



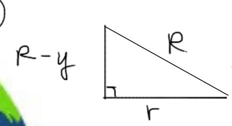
Spherical Volume, radius = R

① Strategy: $V = \int_0^h \text{area}\, dy \overset{⑥}{=} \int_0^h \pi\left(R^2 - (R-y)^2\right) dy$ ⑤

② Area slice: slices = circles $\Rightarrow$ $A = \pi r^2 = \pi\left(R^2 - (R-y)^2\right)$ ④



③

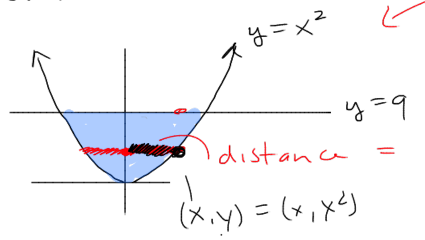


$\Rightarrow$

$(R-y)^2 + (r)^2 = R^2$

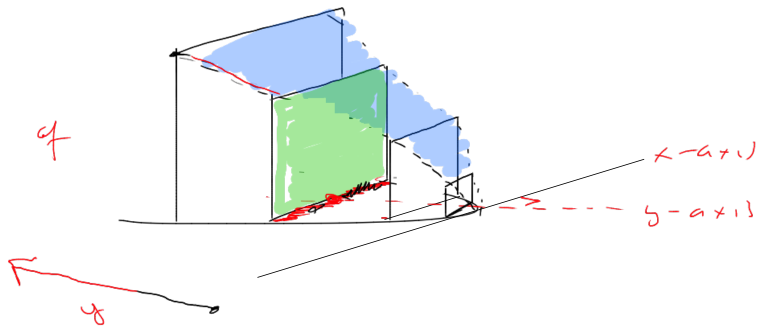
$$\boxed{r = \sqrt{R^2 - (R-y)^2}}$$

6.2.5.



distance = x-coord of $y = x^2$

← base of a solid whose <u>cross-sections</u> are <u>squares</u>.



What's Volume?

① $V = \int \text{slice area}$.

② Area of Square: $A = s^2$
where s = side length

③ Need formula for s in terms of <u>y</u>

solve $y = x^2$ for x

$x = \sqrt{y}$ gives $\frac{1}{2} s$.

So $A = s^2 = (2\sqrt{y})^2 = 4y$