Winter 2025: Take

Dat 495 · Non-parametric Stats

(MatL elective

data "

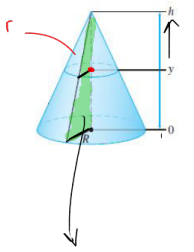
↕

Dat 304

Prerequisites: DAT109 & MA161

6-2-2

$h = 20$
$R = 4$ $\Rightarrow$ what's the Volume?

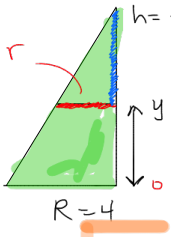


- Strategy: $V = \int$ area of slice
- area of slice?

slices = disks = $A = \pi r^2$

$A = \pi\left[\left(\frac{1}{5}\right)(20 - y)\right]^2 = \frac{\pi}{25}(20 - y)^2$

- Need formula for r, radius of slice: get this from geometry



Similar $\triangle$'s $\Rightarrow$ $\frac{\text{short leg}}{\text{long leg}} = \text{constant} = \frac{4}{20} = \frac{r}{20 - y}$

isolate r $\rightarrow$ $r = \frac{4(20 - y)}{20}$

$\boxed{r = \frac{1}{5}(20 - y)}$

$V = \int_0^{20} \frac{\pi}{25}(20 - y)^2 \, dy = \int \frac{\pi}{25}(u)^2(-du) = -\frac{\pi}{25}\int_{20}^{0} u^2 \, du = \frac{-\pi}{25} \cdot \frac{u^3}{3}\Big|_{20}^{0} = \frac{-\pi}{25}\left[0 - \frac{20^3}{3}\right]$

$= \frac{\pi}{25} \cdot \frac{20^3}{3}$

$u = 20 - y$

$du = -dy$

$-du = dy$

the "u" corresp to $y = 0$ use, set $y = 0$

$u = 20 - 0$

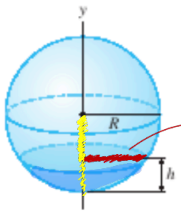
$= 20$

$\frac{\pi}{5 \cdot 5} \cdot \frac{20 \cdot 20 \cdot 20}{3}$

$= \frac{320\pi}{3}$

6-2-4 Strategy: get area of slice & integrate wrt y from y=0 to y = R/4 of circle @ height y,

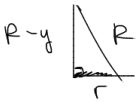
Find the volume of liquid needed to fill a sphere of radius R to height $h = \frac{R}{4}$.



① $A = \pi r^2$, need formula for radius in terms of y.

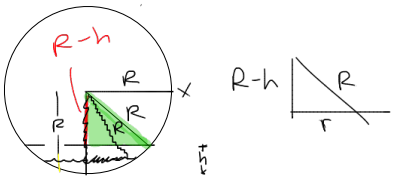
radius r of a slice

For a slice @ height y above bottom



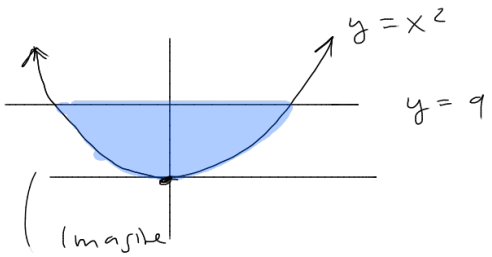
$(R-y)^2 + r^2 = R^2$

$$r = \sqrt{R^2-(R-y)^2}$$

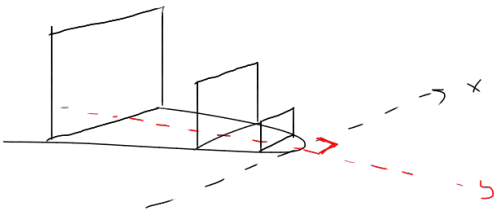


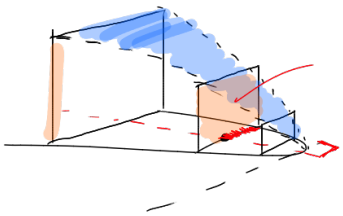
Proceed as usual $= V = \int_0^h \pi r^2 = \int_0^h \pi \left(\sqrt{R^2-(R-y)^2}\right)^2 = \int_0^h \pi \left(R^2-(R-y)^2\right) dy$

6-2-5



Imagine this is the base of a solid with square cross-sections





Area of Square!
the side length is twice the x-coord of $y = x^2$, so $x = \sqrt{y}$
$\ell = 2\sqrt{y}$