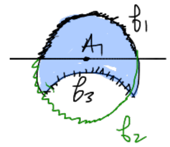
Wed. Wk 12

warm-up:

Set up, in Desmos, an integral to calculate the area of an eclipsed circle.



circle:

$(x-h)^2 + (y-k)^2 = r^2$

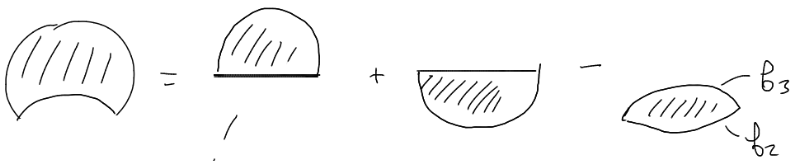
solve for y

$(y-k)^2 = r^2 - (x-h)^2$

$y = \pm\sqrt{r^2-(x-h)^2} + k$

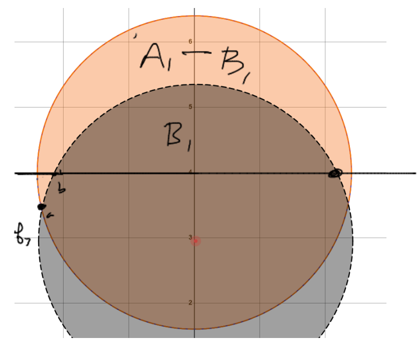
$+$ branch gives f_1

$-$ branch gives f_2



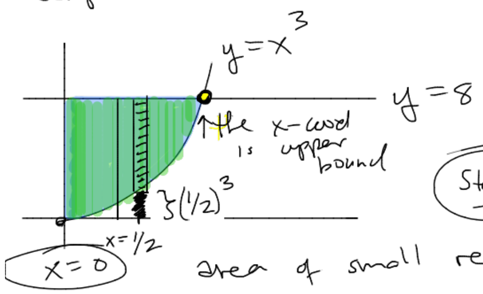
ht of center

$= \int f_1 - \# \quad + \int \# - f_2 \quad - \int f_3 - f_2$



$$\int_{-2.4}^{2.4} \sqrt{1-(x-1)^2} + 4 - \sqrt{1-(x-2)^2} + 3 \, dx$$

Computing Area

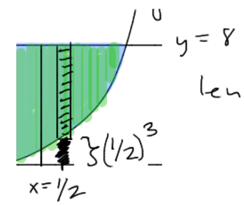


Find the shaded area

Step 1

area of small rectangle = length × width (height)

$$= (8-x^3)\,\Delta x$$



length of rectangle $= 8-(1/2)^3$

Step 2

Bounds of Integration

$x=0$ is lower bound

$x^3=8$ gives upper bound

$\Rightarrow x=2$

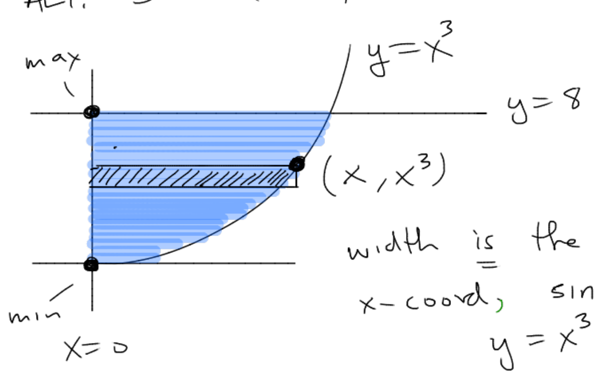
Step 3 Build Integral

$$\int_0^2 \text{Height}\,dx = \boxed{\int_0^2 8-x^3\,dx}$$

$$= 8x-\frac{x^4}{4}\Big|_0^2 = 8\cdot 2-\frac{16}{4}-\left[8\cdot 0-\frac{0^4}{4}\right]$$

$$= 16-4 = 12 \text{ sq. units}$$

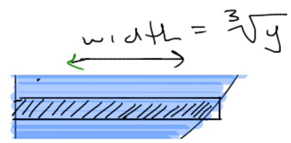
ALT. STRATEGY



width is the x-coord, since $y=x^3$

solve for x:

$\sqrt[3]{y}=x$



height $=\Delta y$

Area Rectangle

Key: (x,y) — up/down dist; the x-coord measures right/left dist.

① $\sqrt[3]{y}\cdot\Delta y$

② Bounds:

min y — max y

$= 0$ — $= 8$

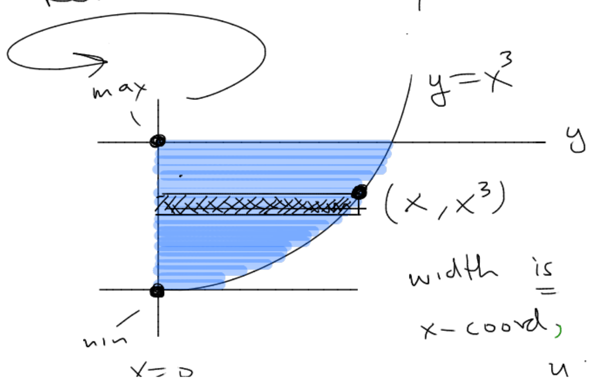
③ $$\int_0^8 \sqrt[3]{y}\,dy = \int_0^8 y^{1/3}\,dx = \frac{3}{4}y^{4/3}\Big|_0^8 = \frac{3}{4}(8)^{4/3}-0$$

$$= \frac{3}{4}\left[(8)^{1/3}\right]^4$$

$$= \frac{3}{4}(2)^4 = \frac{3}{4}\cdot 16 = 12$$

Volume = Sum of Smaller / Simpler Volumes

Revolve about y-axis = Solid of Revolution



$$\text{Vol} = \int \underline{\text{volume of small disks}}\, dy$$

$$\boxed{\int_0^8 \pi\left(\sqrt[3]{y}\right)^2 dy \quad = V}$$

Area disk Δy

$$\pi r^2 \Delta y = \pi\left(\sqrt[3]{y}\right)^2 \Delta y$$