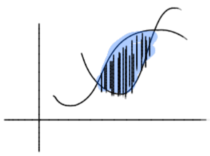
6.2 wk 12 – Wed

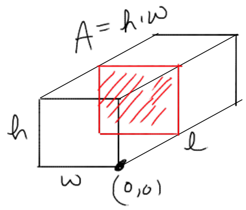
Data 495 (pre-req Dat 109, MA161)

Big Idea: Integration ~ Infinite Addition

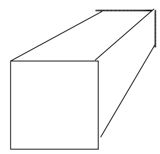


add smaller areas

$\int_a^b \overbrace{f(x) - g(x)}^{\text{height}}\,dx$



$V = l \cdot w \cdot h$



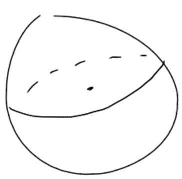
$V = \int_a^b \text{Area}\, dx = \int_0^l h \cdot w \, dx = hw \int_0^l 1\,dx$

$= hw(x)\Big|_0^l$

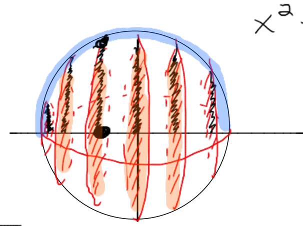
$= hwl - hw0$

$= hwl$

Sphere:



$V = \frac{4}{3}\pi r^3$



$x^2 + y^2 = r^2$

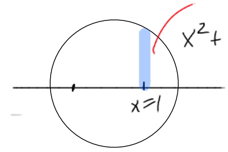
sum of volume of each slice gives total

Area of each small circular slice

radius of each small slice is $\sqrt{r^2 - x^2}$

$y = \pm\sqrt{r^2 - x^2}$

$\sqrt{r^2 - 1^2}$



$\pi R^2 = \pi\left(\sqrt{r^2 - x^2}\right)^2$

$= \pi(r^2 - x^2)$

$V = \int_{\min x}^{\max x} \text{Area}\, dx \overset{(\star)}{=} \int_{-r}^{r} \pi(r^2 - x^2)\,dx = \pi\left[r^2 x - \frac{x^3}{3}\right]\Big|_{-r}^{r}$ $\qquad \frac{3}{3}\cdot 2 - \frac{2}{3}$

$= \pi\left[r^2(r) - \frac{r^3}{3} - \left(r^2(-r) - \frac{(-r)^3}{3}\right)\right] = \pi\left[r^3 - \frac{r^3}{3} + r^3 - \frac{r^3}{3}\right] = \pi\left[2r^3 - \frac{2r^3}{3}\right]$

$= \frac{4\pi r^3}{3}$

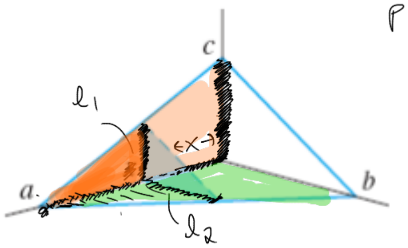
(★) "Even" Property for Integrals

$\int_{-a}^{a} \text{even function} = 2\int_0^a \text{even function}$

$= 2\int_0^r \pi(r^2 - x^2)\,dx$

$2\left[\pi r^2 x - \pi\frac{x^3}{3}\Big|_0^r\right] = 2\left[\pi r^3 - \pi\frac{r^3}{3}\right] = \frac{4\pi r^3}{3}$

Assume that $a = 6$, $b = 4$, and $c = 3$. Volume of Prism (tetra-hedron)



Plan: compute area of cross-section (triangle) $A = \frac{1}{2} b \cdot h$

① use ℓ_1, ℓ_2 e.g. $A = \frac{1}{2} \ell_1 \cdot \ell_2$

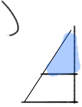
② Integrate wrt x

Need: formulas for ℓ_1 & ℓ_2 (use geometry)

e.g., Similar Δ's

what: same angles

so what? ratio of corresp sides are equal.



ac - Δ

$$\frac{\text{short leg}}{\text{long leg}} = \frac{c}{a} = \frac{\ell_1}{a - x}$$

ab - Δ

$$\frac{b}{a} = \frac{\ell_2}{a - x}$$

Plan from here;
sub values for a, b, c $\rightsquigarrow$ isolate ℓ_1, ℓ_2 • get area of slice $= \frac{1}{2} \ell_1 \ell_2$, integrate from 0 to a.

Reminder: u-sub + definite integrals

you "could" do

$$\int_{-3}^{3} (1-x)^2\,dx = \int_{-3}^{3} 1-2x+x^2\,dx = x - x^2 + \frac{x^3}{3}\Bigg|_{-3}^{3}$$

$$= 3 - 3^2 + \frac{3^3}{3} - \left(-3 - (-3)^2 + \frac{(-3)^3}{3}\right) = 24$$

$3 - 9 + 9 \quad +3 \quad +9 + 9$

$u = 1 - x$

$\frac{du}{dx} = -1$

$-du = dx$

$x = -3 \Rightarrow u = 1-(-3) = 4$

$x = 3 \Rightarrow u = 1 - 3 = -2$

$$-\int_{4}^{-2} u^2\,du = -\frac{u^3}{3}\Bigg|_{4}^{-2} = -\frac{(-2)^3}{3} - \left(-\frac{4^3}{3}\right)$$

$$= \frac{8}{3} + \frac{64}{3} = \frac{72}{3} = 24$$