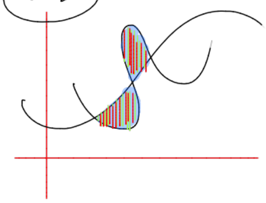
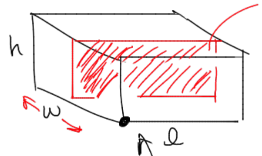
6-2



slice →

get cross-sections | add 'em up

Volume



Area of Cross Section = hl

$V = l \cdot w \cdot h$

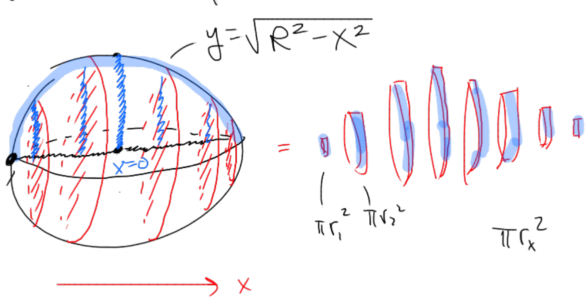
set as $(0,0)$ integrate from 0 to w :

$$\int_0^w \text{Area}\, dy$$

$$\int_0^w hl\, dy = hl \int_0^w 1\, dy = hly\Big|_0^w = hlw - hl0 = lwh$$

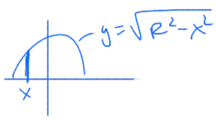
constant!

Volume of a Sphere w/ radius R : $\frac{4}{3}\pi R^3$



$x^2+y^2=R^2$

$y = \pm\sqrt{R^2-x^2}$



Area of Circle: $A = \pi r^2$ | what controls the radius? $r = \sqrt{R^2-x^2}$

$$\int_{\min x}^{\max x} \text{Area of Slice}\, dx = \int_{-R}^{R} \pi\left(\sqrt{R^2-x^2}\right)^2 dx = \int_{-R}^{R} \pi\,(R^2-x^2)\,dx$$

$$= \pi\left[R^2x - \frac{x^3}{3}\right]\Bigg|_{-R}^{R} = \pi\left[\left[R^2\cdot R - \frac{R^3}{3}\right] - \left[-R^3 + \frac{R^3}{3}\right]\right] = \pi\left[2R^3 - \frac{2R^3}{3}\right] = \boxed{\frac{4\pi R^3}{3}}$$

$R^3 - \frac{R^3}{3}$

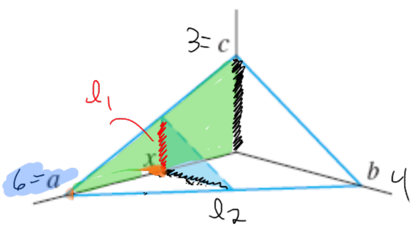
Even SHORTCUT PROPERTY

$$\int_{-a}^{a} \text{even function} = 2\int_0^a \text{even function}$$

$$\int_{-R}^{R} \pi\,(R^2-x^2)\,dx = 2\int_0^R \pi\,(R^2-x^2)\,dx$$

$$= 2\left(\pi\left[R^2x - \frac{x^3}{3}\right]\right]_0^R = 2\pi\cdot\left(R^3 - \frac{R^3}{3} - 0\right) = \frac{4\pi R^3}{3}$$

Assume that $a = 6$, $b = 4$, and $c = 3$.



Solid

Volume of Prism ?

$\frac{1}{2} l_1 \cdot l_2$

$\parallel$

(triangle)

Strategy:

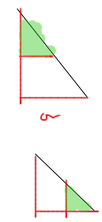
(1) Compute area of cross-section

(2) Integrate that area wrt x

Ratios of corresp. sides are $=$

$\circledast$ = all angles are $=$ $\Rightarrow$

key: use similar Δ's $\circledast$ on two planes on ac-plane & bc-plane



(I) ac-plane

$$\frac{c}{a} = \frac{l_1}{6-x}$$

ab-plane

$$\frac{\text{short leg}}{\text{long leg}} = \frac{4}{6} = \frac{l_2}{6-x}$$

(II) Find l_1, l_2 in terms of x, plug into Area, integrate from $x=0$ to $x=6$. (don't forget u-sub!)

u-sub + definite integrals

$$\int_{-3}^{3}(1-x)^2\,dx \xrightarrow{\text{while it's possible to do this}} \int_{-3}^{3} 1-2x+x^2\,dx = x - x^2 + \frac{x^3}{3}\Big|_{-3}^{3}$$

$$= 3 - 3^2 + \frac{3^3}{3} - \left(-3 - (-3)^2 - \frac{3^3}{3}\right)$$

$$= 3-9+9 - (-3-9-9)$$

$$+3 \quad +3 \quad +18 = 6+18 = 24$$

① $u = 1-x$

$du = -1\,dx$

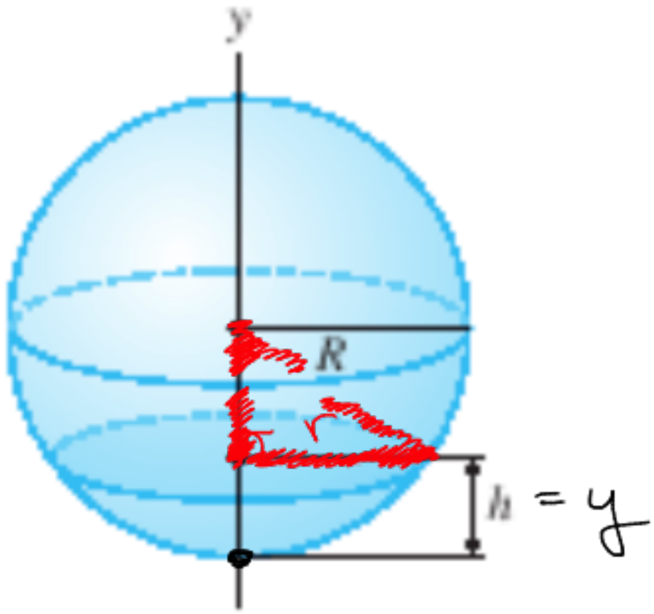
$-du = dx$

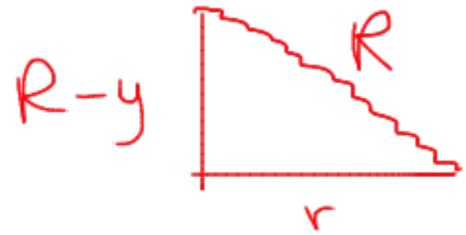
② Change Bounds

$x=-3 \Rightarrow u = 1-(-3) = 4$

$x = 3 \Rightarrow u = 1-3 = -2$

$$\int_4^{-2} u^2 \cdot -du = -\int_4^{-2} u^2\,du = -\frac{u^3}{3}\Big|_4^{-2} = +\frac{8}{3} - \left(-\frac{64}{3}\right) = \frac{72}{3} \approx 24$$


y
R
r
h = y


R
R-y
r