Exam 4 Review

1. If a snowball melts so that its surface area decreases at a rate of 1 square centimeter per minute, find the rate at which the diameter decreases when the diameter is 10 centimaters.

2. Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 radians per second. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.

3. Find an equation of the tangent line to the function at the given point.
(a) $f(x) = e^{x/10}, x = 0$
(b) $f(x) = \sqrt{x}, x = 16$

4. Use your work in #3 find estimates . . ..
(a) $e^{0.1/10}$, $e^{.25/10}$
(b) $\sqrt{16.5}$, $\sqrt{17.5}$

5. A boat leaves a dock at 1:00 PM and travels due south at a speed of 20 kph. Another boat has been heading due east at 12 kph and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

6. A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

7. Evaluate the limit . . . .

(a)

$$\lim_{x \to +\infty} \frac{(\ln x)^2}{x}$$

(b)

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

(c)

$$\lim_{x \to 0} \frac{x + \sin x}{x + \cos x}$$

(d)

$$\lim_{x \to 0} \left(1 - \frac{x}{2}\right)^{1/x}$$

8. Find the absolute maximum and absolute minimum of the function on the indicated interval.

(a) $f(x) = 3x^4 + 8x^3 - 18x^2 + 5$, $[-4, 2]$

(b) $f(x) = 3x^4 - 4x^3 - 12x^2$, $[-3, 1]$

9. Find the average value of the function $\sin x$ on the interval $[0, \pi]$.

10. Find the average value of the function $\frac{1}{\sqrt{1-x^2}}$ on the interval $[0, 0.5]$.

11. Sketch the region enclosed by the given curves. Use an integral to find the area enclosed.
(a) $y = x$, $y = x^2$
(b) $y = x^2 - 2x$, $y = x + 4$

12. Find the volume when the area enclosed in #11(a) is rotated . . .
(a) around the x-axis
(b) around the line $x = -1$
(c) around the y-axis
(d) around the line $y = 2$

13. When a particle is located a distance x meters from the origin, a force of $\dfrac{1}{1+x^2}$ Newtons acts on it. How much work is done in moving the particle from $x = 0$ to $x = 1$?