x-coords are $\equiv 0$

As a hardworking student, plagued by too much homework, you spend all night doing math homework. By 6am, you imagine yourself to be a region bounded by

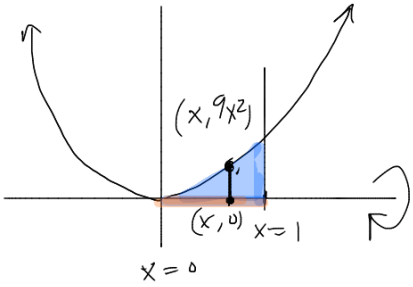
$y = 9x^2$ → ↑↓ or ↻

$x = 0$

$x = 1$

$y = 0$

As you grow more and more tired, the world begins to spin around you. However, according to Newton, there is no difference between the world spinning around you, and you spinning around the world. Unfortunately, you are so tired that you think the world is the x-axis. What is the volume of the solid you (the region) create by spinning about the x-axis?




① Visual:

② Slice ⊥ to axis! Region is "attached" to axis ⇒ Disk

$9x^2 = r$ Center of disk is on axis

③ Area of Circle: $\pi r^2 = \pi (9x^2)^2 = 81\pi x^4$

④ $V = \int_{\min(x)}^{\max(x)} \text{Area}\, dx = \int_0^1 81\pi x^4\, dx$

int. along axis of rev.

# 6

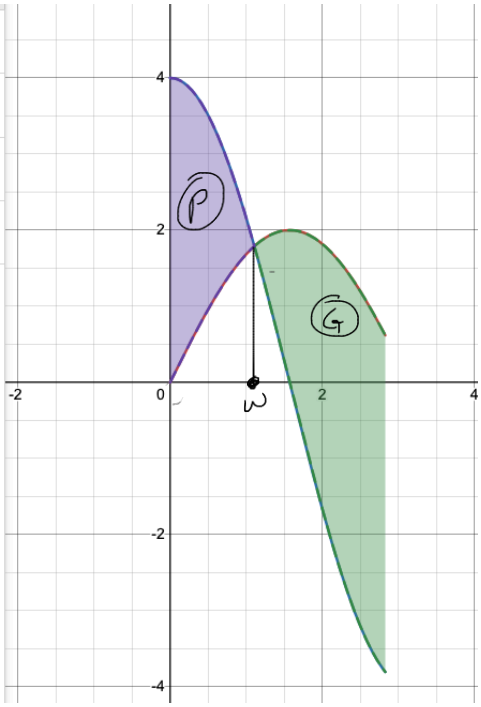
Find the area of the region enclosed between $y = 2\sin(x)$ and $y = 4\cos(x)$ from $x = 0$ to $x = 0.9\pi$.
Hint: Notice that this region consists of two parts.

$f(x) = 2\sin(x)\{0 \le x \le .9\pi\}$

$g(x) = 4\cos(x)\{0 \le x \le .9\pi\}$

$g(x) < y < f(x)$

$f(x) < y < g(x)$




$P = \int_0^w 4\cos x - 2\sin x \, dx$

To find w: solve

$4\cos x = 2\sin x$

$2 = \frac{\sin x}{\cos x} = \tan x$

$\tan^{-1}(2) = x = 1.107$

set $w = 1.107$

$G = \int_w^{.9\pi} 2\sin x - 4\cos x \, dx$

Ans: P + G

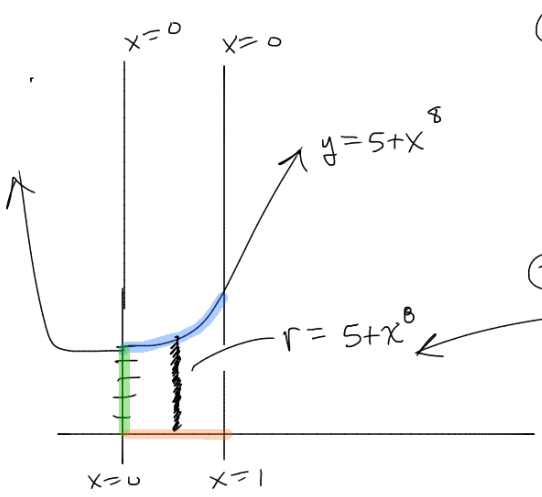
Find the volume of the solid formed by rotating the region enclosed by

$$x = 0,\ x = 1,\ y = 0,\ y = 5 + x^8$$

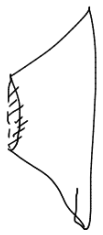
(vertical; $x = 0$: y-axis; $y = 0$: x-axis; $x^{even} \Rightarrow$ ∪)

about the $x$-axis.

Answer:




① virtual



② slice = $A = \pi(5 + x^8)^2$

③ $\int_0^1 \pi(5 + x^8)^2 \, dx$ = algebra 1st

$= \pi \int_0^1 (25 + 10x^8 + x^{16}) \, dx$

$= \pi \left[ 25x + \frac{10x^9}{9} + \frac{x^{17}}{17} \right] \Big|_0^1 = \pi \left[ \left( 25 + \frac{10}{9} + \frac{1}{17} \right) - (0 + 0 + 0) \right]$

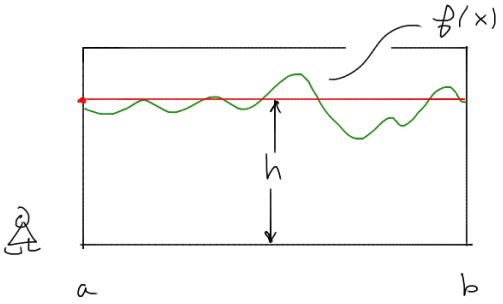
# Average Velocity

eg., Average Value

(1) Recall, how do you find the average of a (finite) set of numbers, .. add 'em up and divide by how many there are

(2) Average Value: find the average of an infinite set of #'s

Idea:




$$\text{Area Under Curve} = \int_a^b f(x)\,dx$$

$$\text{Area of Rectangle} = h \cdot (b-a)$$

avg height, length

so $$h = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\text{Avg Value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex Suppose from $t=1$, to $t=10$ your velocity $v(t) = 50t^2$ $\frac{m}{s}$

the average velocity is

$$\frac{1}{10-1}\int_1^{10} 50t^2\,dt = \frac{1}{9}\left[\frac{50t^3}{3}\right]\Big|_1^{10}$$

$$= \frac{1}{9}\left[\frac{50\cdot 1000}{3} - \frac{50}{3}\right] = \frac{1}{9}(17000)$$

$$\approx 1700\ \tfrac{m}{s}$$

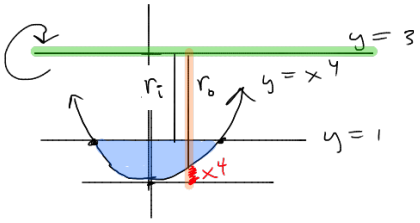
Find the volume of the solid obtained by rotating the region bounded by

$$y = x^4, y = 1;$$

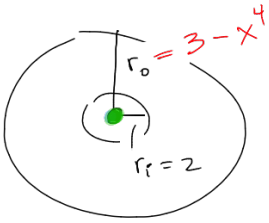
about the line $y = 3$

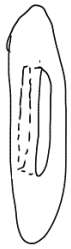
Answer:

① Region / visual




② Slice ⊥ to axis of rev. ⇒ vertical slice



"Thick tire"