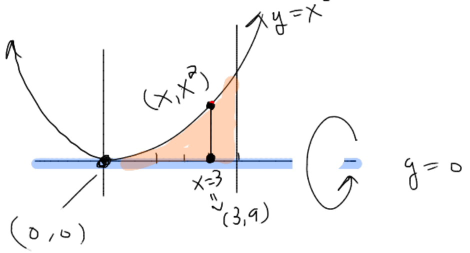
Volume 6-2, 6-3, 6-4

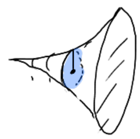
6-2 Compute volumes of solids via integrating their cross-sections.

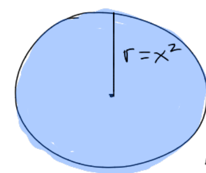
6-3 (same idea) } solids of Revolution
6-4 "

Idea Rotate the region bound by $y=0$, $y=x^2$, $x=4$ about x-axis. Compute it's volume.

Radius: segment touching axis & graph on its ends







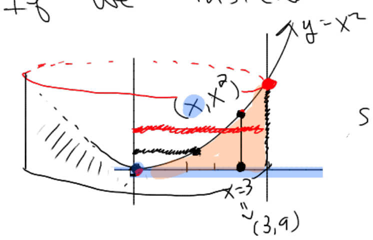
$A=\pi r^2=\pi(x^2)^2=\pi x^4$

Integrate ALONG axis of revolution.

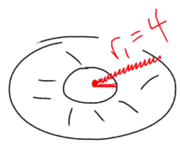
$$V=\int_{\text{min } x}^{\text{max } x} \text{Area } dx = \int_0^4 \pi x^4\, dx = \pi\frac{x^5}{5}\Big|_0^4 = \boxed{\frac{1024\pi}{5} \text{ cubic units}}$$

If we instead revolve about y-axis:

Axis of Rev = Vertical (y-axis) $\Rightarrow$ Int. wrt. y.



slice $\longrightarrow$



Missing Inner Circle

$r_2=\sqrt{y}$

$y=x^2$

$\sqrt{y}=x$ *

Area $=\pi r_1^2-\pi r_2^2=\pi(16-y)$

$$V=\int_{\text{min } y}^{\text{max } y}\pi(16-y)\,dy = \pi\int_0^{16} 16-y\,dy = \pi\left(16y-\frac{y^2}{2}\right)\Big|_0^{16} = \pi\left(16^2-\frac{16^2}{2}\right)=\pi(2^8-2^7)$$

$=128\pi$ cubic units

$.75\cdot(\text{Exam Avg}) + .25(\text{Achieve}) = \text{Estimated Grade}$

Name: ______________________________ **Exam 3** November 9, 2024

Find the indicated antiderivative. Show your work

1. $\displaystyle\int \frac{5x^4}{\sqrt{x^5-1}}\,dx =$

2. $\displaystyle\int 3x(\overset{u}{\sin(x^2)+1})^2\cos(x^2)\,dx =$

$u = \sin(x^2)+1$

$du = \cos(x^2)\cdot 2x\,dx$

$\frac{1}{\cos(x^2)\cdot 2x}\,du = dx$

$= 3\int \cancel{x}\cdot(u)^2\,\cancel{\cos(x^2)}\cdot\frac{1}{\cancel{\cos(x^2)}\cdot 2\cancel{x}}\,du$

$= \frac{3}{2}\int u^2\,du = \frac{3}{2}\frac{u^3}{3} = \frac{u^3}{2} + C = \boxed{\frac{(\sin(x^2)+1)^3}{2} + C}$

OR

$u = x^2 \Rightarrow du = 2x\,dx,\quad \frac{1}{2x}\,du = dx$

$= 3\int \cancel{x}(\sin(u)+1)^2\cdot\cos(u)\frac{1}{2\cancel{x}}\,du$

$= \frac{3}{2}\int(\sin(u)+1)^2\cdot\cos(u)\,du$

do another u-sub

$w = \sin(u)+1$

$dw = \cos(u)\,du$

$= \frac{3}{2}\int w^2\,dw = \frac{\cancel{3}}{2}\frac{w^3}{\cancel{3}} + C = \frac{w^3}{2} + C$

$= \frac{(\sin(x^2)+1)^3}{2} + C$

3. $\displaystyle\int \frac{\cos(x)}{e^{\sin(x)}}\,dx = \overset{\text{think}}{\int e^u\,du}$

$\displaystyle\int e^{-\sin x}\cdot\cos x\,dx = \int e^u\cos x\left(-\frac{1}{\cos x}\right)du$

$u = -\sin x$

$du = -\cos x\,dx$

$\frac{-1}{\cos x}\,du = dx$

$= -\int e^u\,du = -e^u + C$

$= \boxed{e^{-\sin x} + C}$

4. $\displaystyle\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx =$

5. $\displaystyle\int \frac{2x-2}{x^2+1}\,dx =$

eg $\displaystyle\int (5-x)^{3/2}\cdot x\,dx$

6. $\displaystyle\int x\sqrt{x-7}\,dx = \int x\,(x-7)^{\frac{1}{2}}\,dx$

$u = x-7$

$du = dx$

$u+7 = x$

$= \displaystyle\int x \cdot u^{1/2}\,du$ mixed variables

$= \displaystyle\int (u+7)\cdot u^{1/2}\,du$

$= \displaystyle\int u^{3/2} + 7u^{1/2}\,du = \frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}\cdot 7\cdot u^{\frac{3}{2}} + C = \boxed{\frac{2}{5}(x-7)^{\frac{5}{2}} + \frac{14}{3}(x-7)^{\frac{3}{2}} + C}$

7. $\displaystyle\int \frac{x^2+x+1}{4x}\,dx = \int \frac{x^2}{4x} + \frac{x}{4x} + \frac{1}{4x}\,dx$

$= \displaystyle\frac{1}{4}\int x\,dx + \frac{1}{4}\int 1\,dx + \frac{1}{4}\int \frac{1}{x}\,dx$

$= \displaystyle\frac{\frac{1}{4}x^2}{2} + \frac{1}{4}x + \frac{1}{4}\ln|x| + C$

$= \boxed{\dfrac{x^2}{8} + \dfrac{x}{4} + \dfrac{\ln|x|}{4} + C}$

8. $\displaystyle\int 8x^3\sqrt{x^4-1}\,dx =$

9. $\displaystyle\int \frac{4x}{\sqrt{1-x^4}}\,dx =$

10. $\displaystyle\int \tan^2(x)\sec^2(x)\,dx =$

11. $\int \frac{\ln(2x)}{x}\,dx = \int \ln(2x) \cdot \frac{1}{x}\,du$

$u = \ln(2x)$

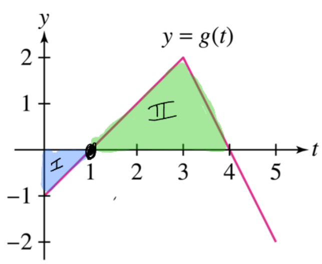
$(\ln kx)' = (\ln k + \ln(x))' = (\ln x)' = \frac{1}{x}$

$\ln k \downarrow$ constant

12. Use the figure to evaluate $\int_0^4 g(t)\,dt = \int_0^1 g(t)\,dt + \int_1^4 g(t)\,dt$

signed area of I (negative)

area of II



$-\frac{1}{2}(1)(1) + \frac{1}{2}(3)(2) = -\frac{1}{2} + 3 = 2.5$

13. Let $f''(x) = 6x + 4$ and assume $f'(2) = 5$ and $f(1) = 10$. Determine $f(x)$.

$f'(x) = \int f''(x)\,dx = \int 6x + 4\,dx = \frac{6x^2}{2} + 4x + C$

sub $x = 2$

$f'(2) = 5 = 3(2)^2 + 4(2) + C$ so $5 = 12 + 8 + C$ $C = -15$

$\boxed{f'(x) = 3x^2 + 4x - 15}$

$f(x) = \int f'(x)\,dx = \int 3x^2 + 4x - 15\,dx = x^3 + 2x^2 - 15x + D$

$f(1) = 10 = 1^3 + 2(1) - 15 + D \Rightarrow D = +22$

$\boxed{f(x) = x^3 + 2x^2 - 15x + 22}$

14. Fill in the blank. One thing I think I'll remember from this class is ____________________________________

u-sub w/ definite integrals

$$\int_{-1}^{2} (2x+1)^3\,dx = \int_{-1}^{5} u^3 \tfrac{1}{2}\,du = \tfrac{1}{2}\cdot\frac{u^4}{4}\Big|_{-1}^{5} = \frac{u^4}{8}\Big|_{-1}^{5} = \frac{5^4}{8} - \frac{(-1)^4}{8} = \frac{5^4-1}{8}$$

same

$$\frac{(2x+1)^4}{8}\Big|_{-1}^{2} = \frac{(2\cdot2+1)^4}{8} - \frac{(2(-1)+1)^4}{8}$$

$u = 2x+1$

$du = 2dx \Rightarrow \tfrac{1}{2}du = dx$

when $x=-1$, $u = 2(-1)+1 = -1$

$x = 2$, $u = 2(2)+1 = 5$

$$\int_{-1}^{1} (2x+1)^2\,dx = \int u^2 \cdot \tfrac{1}{2}\,du = \tfrac{1}{2}\frac{u^3}{3} = \tfrac{1}{6}(2x+1)^3 \Big|_{-1}^{1}$$

$u = 2x+1$

$du = 2\ dx$

$\tfrac{1}{2}du = dx$

Volume: applications of integration

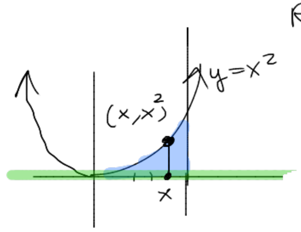
"solids of revolution"

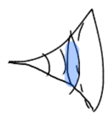
6-2, 6-3, 6-4
(more of same)

↓

computing volume by integrating cross-sectional area

Revolve region bounded by about x-axis (horizontal)

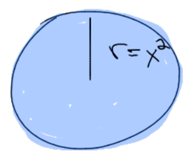




by: $y=x^2$
$y=0$
$x=4$

radius = segment w/ axis on end & curve on other
integrate w.r.t. axis of revolution (horizontal ⇒ x)

$$V=\int_{\min x}^{\max x} \text{Area}\,dx$$



$$= \int_0^4 \pi r^2 dx = \int_0^4 \pi x^4 dx = \frac{\pi x^5}{5}\Big|_0^4 = \frac{\pi \cdot 4^5}{5} = \boxed{\frac{1024\pi}{5} \text{ unit}^3}$$