Name: ______________________ **Exam 3** November 9, 2024

Find the indicated antiderivative. Show your work

1. $\displaystyle\int \frac{5x^4}{\sqrt{x^5-1}}\,dx =$

2. $\displaystyle\int 3x(\sin(x^2)+1)^2\cos(x^2)\,dx =$ $3\int \cancel{x}(u)^2 \cancel{\cos(x^2)} \frac{1}{\cancel{\cos(x^2)}\cdot 2\cancel{x}}\,du = \frac{3}{2}\int u^2\,du$

$u = \sin(x^2)+1$

$du = \cos(x^2)\cdot 2x\,dx$ $= \frac{3}{2}\frac{(\sin(x^2)+1)^3}{3} + c = \boxed{\frac{\sin(x^2+1)^3}{2}}$

$\frac{1}{\cos(x^2)\cdot 2x}\,du = dx$

3. $\displaystyle\int \frac{\cos(x)}{e^{\sin(x)}}\,dx =$

4. $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx =$

5. $\int \frac{2x-2}{x^2+1}\,dx =$

6. $\int x\sqrt{x-7}\,dx =$

7. $\int \frac{x^2+x+1}{4x}\,dx = \int \frac{x^2}{4x} + \frac{x}{4x} + \frac{1}{4x}\,dx$

$= \int \frac{x}{4} + \frac{1}{4} + \frac{1}{4x}\,dx = \frac{1}{4}\int x\,dx + \frac{1}{4}\int dx + \frac{1}{4}\int \frac{1}{x}\,dx$

$= \frac{x^2}{8} + \frac{1}{4}x + \frac{1}{4}\ln|x| + C$

8. $\int 8x^3\sqrt{x^4-1}\,dx =$ $\int 8x^3 (x^4-1)^{\frac{1}{2}}\,dx = 8\int \cancel{x^3}\cdot u^{\frac{1}{2}}\cdot \frac{1}{4\cancel{x^3}}\,du = 2\int u^{\frac{1}{2}}\,du$

$u = x^4-1$

$du = 4x^3\,dx$

$\frac{1}{4x^3}\,du = dx$

$$2\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \boxed{\frac{4}{3}(x^4-1)^{3/2} + c}$$

9. $\int \frac{4x}{\sqrt{1-x^4}}\,dx =$

10. $\int \tan^2(x)\sec^2(x)\,dx =$ $\int u^2\,du = \frac{u^3}{3}+c$

$u = \tan(x)$

$du = \sec^2 x\,dx$

$$\frac{\tan^3(x)}{3} + c$$

check: $\frac{\cancel{3}\cdot\tan^2(x)}{\cancel{3}}\cdot\sec^2 x$

$(\ln(kx))' = (\ln k + \ln x)'$

$(\ln k)' + (\ln x)'$

$= 0 \qquad = \frac{1}{x}$

11. $\displaystyle\int \frac{\ln(2x)}{x}\,dx = \int \underbrace{\ln(2x)}_{u}\cdot\underbrace{\frac{1}{x}\,dx}_{du}$

$u = \ln(2x)$

$du = \frac{1}{2x}\cdot 2\;dx$

$= \frac{1}{x}\,dx$

$x\,du = dx$

$\displaystyle\int \frac{u}{x}\cdot x\,du = \int u^1\,du = \frac{u^2}{2} + c$

$$\boxed{\frac{(\ln(2x))^2}{2} + C}$$

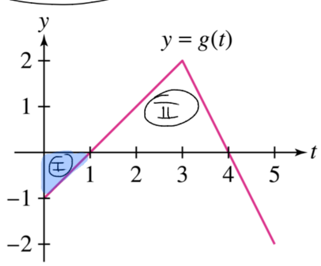
check:

$\frac{1}{2}\left[2(\ln(2x))'\right]\cdot\frac{1}{x}$

$\frac{\ln(2x)}{x}$

☺

12. Use the figure to evaluate $\displaystyle\int_0^4 g(t)\,dt = \int_0^1 g(t)\,dt + \int_1^4 g(t)\,dt$



$\int_0^1 g(t)\,dt \downarrow$ "signed" area of I

$\int_1^4 g(t)\,dt \downarrow$ area of II

$-\frac{1}{2}(1)(1) + \frac{1}{2}(3)(2)$

$-\frac{1}{2} + 3 = 2.5$

13. Let $f''(x) = 6x + 4$ and assume $f'(2) = 5$ and $f(1) = 10$. Determine $f(x)$.

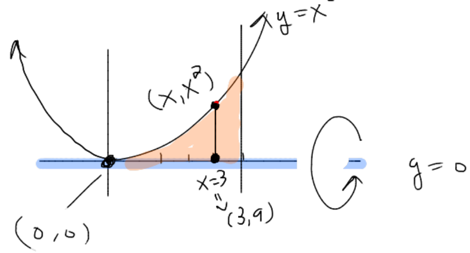
14. Fill in the blank. One thing I think I'll remember from this class is ____

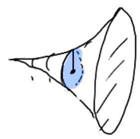
Volume 6-2, 6-3, 6-4

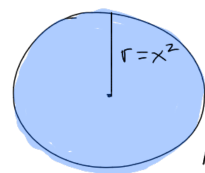
6-2 Compute volumes of solids via integrating their cross-sections.

6-3 (same idea) } solids of Revolution
6-4 "

Idea Rotate the region bound by $y=0$, $y=x^2$, $x=4$ about x-axis. Compute it's volume.







Radius: segment touching axis & graph on its ends

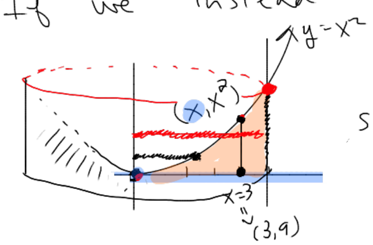
$A=\pi r^2=\pi(x^2)^2=\pi x^4$

Integrate ALONG axis of revolution.

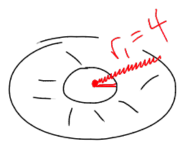
$$V=\int_{\min x}^{\max x} \text{Area}\,dx = \int_0^4 \pi x^4\,dx = \pi\frac{x^5}{5}\Big|_0^4 = \boxed{\frac{1024\pi}{5} \text{ cubic units}}$$

If we instead revolve about y-axis:

Axis of Rev = Vertical (y-axis) ⇒ Int. wrt. y.



slice →



Missing Inner Circle

$r_2=\sqrt{y}$

$\text{Area}=\pi r_1^2-\pi r_2^2=\pi(16-y)$

$y=x^2$ *

$\sqrt{y}=x$

$$V=\int_{\min y}^{\max y}\pi(16-y)\,dy = \pi\int_0^{16} 16-y\,dy = \pi\left(16y-\frac{y^2}{2}\right)\Big|_0^{16} = \pi\left(16^2-\frac{16^2}{2}\right) = \pi(2^8-2^7)$$

$= 128\pi$ cubic units