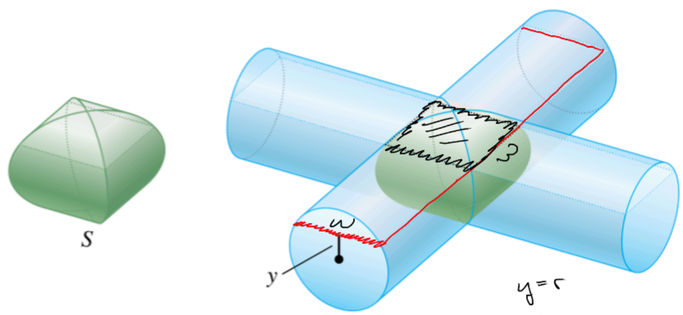
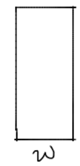
Thu wk 13

6.2.9

The solid S in the figure is the intersection of two cylinders of radius r whose axes are perpendicular.



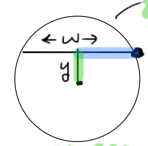
$y = r$
$\Rightarrow w = 0$



check:

$y = 0 \Rightarrow$
$w = 2\sqrt{r^2 - 0^2}$
$= 2\sqrt{r^2}$
$= 2r$

$x^2 + y^2 = r^2$



(x, y)

$w = 2x$

$x = \sqrt{r^2 - y^2}$

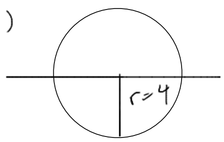
$w = 2\sqrt{r^2 - y^2}$

(a) The horizontal cross section of each cylinder at distance y from the central axis is a rectangular strip. Find the strip's width.

(b) area of cross-section

$A = w \cdot l = w^2$

(c)

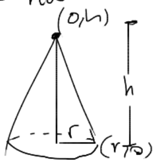


$\int_{-4}^{4} \text{Area of Slice} = \int_{-4}^{4} w^2 \, dy$

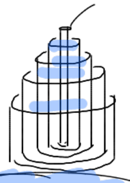
Volumes of Solids of Revolution

Compute Volume of Circular Cone

6.2.1 - hw



=



$V = \frac{1}{3}\pi r^2 h$

$y = -\frac{h}{r}x + h$

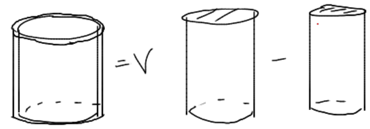
h depends on x by

$\int_0^r 2\pi x h \, dx =$

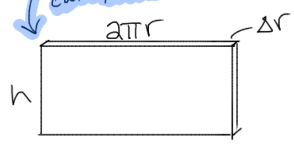
integrate to get volume

$$V = \int_0^r 2\pi \cdot x \cdot f(x) \, dx$$

Vol. of Cylindrical Shell



cut & unroll



$= 2\pi r h \cdot \Delta r$

difference of solid cylinders

$= \pi r_1^2 h - \pi r_2^2 h$

$= \pi h(r_1^2 - r_2^2)$

here $r_2 = r_1 - \Delta r_1$

$= \pi h(r_1^2 - (r_1 - \Delta r_1)^2)$

$= \pi h(\underbrace{r_1^2 - r_1^2}_{=0} + 2r_1 \cdot \Delta r_1 - \underbrace{\Delta r_1^2}_{\approx 0})$ (2nd order)

$= \pi h(2r_1 \Delta r_1)$

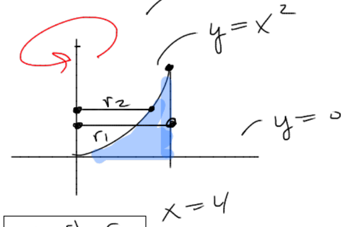
$= 2\pi r_1 h \cdot \Delta r_1$

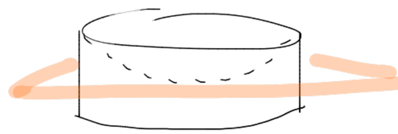
Same

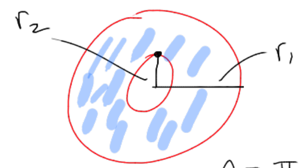
$$V = \int_0^r 2\pi x\left(\frac{-h}{r}x + h\right) dx = 2\pi h \int_0^r -\frac{x^2}{r} + x \, dx = 2\pi h\left(\frac{-x^3}{3r} + \frac{x^2}{2}\right)\Bigg|_0^r$$

$$= 2\pi h\left(\frac{-r^3}{3r} + \frac{r^2}{2}\right) = 2\pi h\left(\frac{-r^2}{3} + \frac{r^2}{2}\right) = 2\pi\left[\frac{-2r^2 + 3r^2}{6}\right] = \frac{\pi r^2}{3}$$

Yesterday's revolve about y-axis







$A = \pi r_1^2 - \pi r_2^2$

③ $= \pi(r_1^2 - r_2^2)$

Washer Method

① slice ⊥ to axis of revolution ⟹ washer

② compute radii : (radius = segment connects axis & curve)

$r_1 = 4$

$r_2 =$ x-coord of $(x,y) = (x, x^2)$ so $y = x^2$

$= \sqrt{y}$ $\Rightarrow \sqrt{y} = x$

it depends on y!

④ Integrate w.r.t. (along) axis ↕ ⟹ y

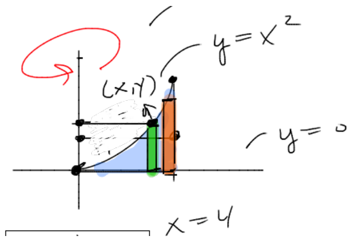
$$\int_0^{\max y = 16} \text{Area}\, dy = \int_0^{16} \pi(4^2 - (\sqrt{y})^2)\, dy = \pi \int_0^{16} 16 - y\, dy$$

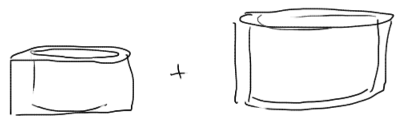
$$= \pi\left(16y - \frac{y^2}{2}\right)\Big|_0^{16} = \pi\left(2^8 - \frac{2^8}{2}\right)$$

$$= \pi(2^8 - 2^7)$$

$$= 2^7\pi = 128\pi$$

Shell Method





$$2\pi\int_0^4 x\cdot \underbrace{f(x)}_{=x^2}\,dx = 2\pi\int_0^4 \underbrace{x\cdot x^2}_{=x^3}\,dx = \frac{\cancel{2}\pi\cdot x^4}{\cancel{4}}\Bigg|_0^4 = \frac{\pi}{2}\cdot(4)^4 = \frac{\pi\cdot 2^8}{2} = 2^7\cdot\pi$$

128π