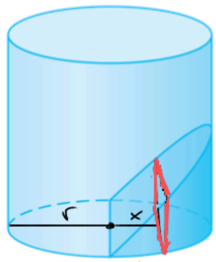
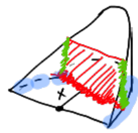
6.2.8

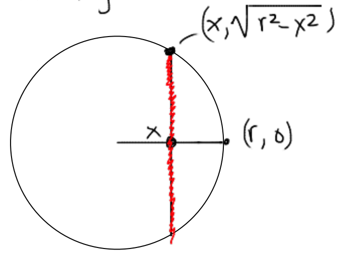
A plane inclined at an angle of 45° passes through a diameter of the base of a cylinder of radius r.

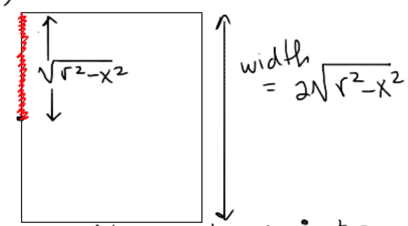


← bottom is circle w/ radius r.



$x^2 + y^2 = r^2$





height of rectangle just = x

Write an expression for the volume V of the region within the cylinder and below the plane in terms of r.

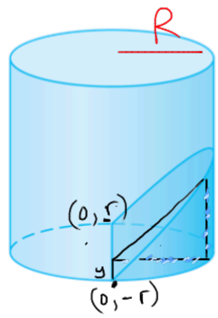
$$\int_0^r \text{Area}\,dx = \int_0^r 2\sqrt{r^2-x^2}\cdot x\,dx = -\int_0^r \sqrt{r^2-x^2}\,(-x\,dx) = -\int_{r^2}^0 \sqrt{u}\,du$$

$u = r^2 - x^2$	$x = 0 \Rightarrow u = r^2$
$du = -2x\,dx$	$x = r \Rightarrow u = 0$

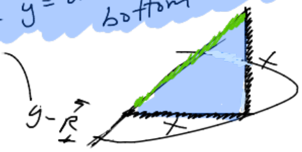
$$= -\frac{2}{3}u^{3/2}\Big|_{r^2}^{0} = \frac{2}{3}(r^2)^{3/2} = \frac{2}{3}r^3$$

6.2.8

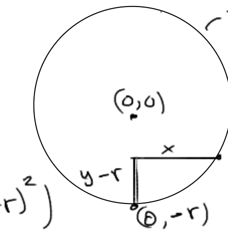
A plane inclined at an angle of 45° passes through a diameter of the base of a cylinder of radius r.



here y = distance from bottom edge of cylinder



Base of triangle is x-coord for height $y-r$



$x^2+y^2=r^2$

$x=\sqrt{r^2-y^2}$

$x=\sqrt{r^2-(y-r)^2}$

$A=\frac{1}{2}\text{base} * \text{height}$

$= \frac{1}{2}x \cdot x = \frac{1}{2}x^2 = \frac{1}{2}\left(r^2-(y-r)^2\right)$

Write an expression for the volume V of the region within the cylinder and below the plane in terms of r.

$$\int_{-r}^{r} \frac{1}{2}\left(r^2 - \overbrace{(y-r)^2}^{-y^2+2yr-r^2}\right) dy = \frac{1}{2}\int_{-r}^{r} 2yr - y^2\, dy \overset{\text{even}}{=} \int_0^r 2yr - y^2\, dy = ry^2 - \frac{y^3}{3}\Big|_0^r$$

$$= r^3 - \frac{r^3}{3} = \frac{2r^3}{3}$$

Volume: method of Shells

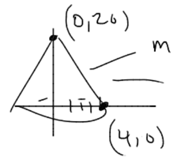
$$\boxed{V = 2\pi \int_a^b x \cdot f(x)\, dx}$$

6.2.11:

$V = \frac{1}{3}\pi r^2 h$ via

6.2.11,
$h = 20$
$r = 4$

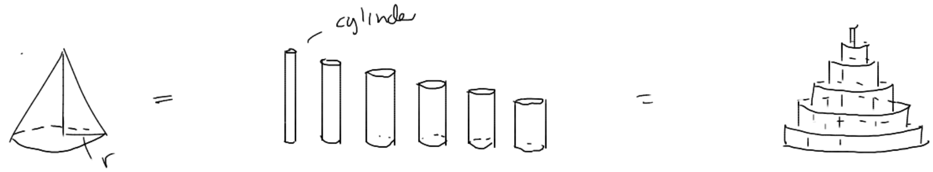
$V = \int_0^{20} 2\pi \cdot x \cdot (-5x+20)$



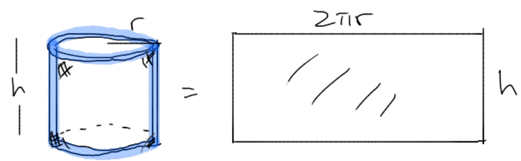
$m = \frac{20-0}{0-4} = -5$

$y = -5x + 20$ — controls height of nested Russian doll cylinder

Shell method



Volume of Cylindrical Shell



$A = 2\pi r h$

$V = 2\pi r h \cdot \Delta r$

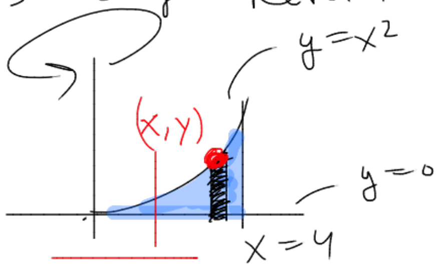
$$V = \int_0^4 2\pi \cdot x \cdot (-5x+20) = 2\pi \int_0^4 -5x^2 + 20x = 2\pi\left(-\frac{5x^3}{3} + \frac{20x^2}{2}\right)\Big|_0^4$$

$$= 2\pi \cdot 5x^2\left[-\frac{x}{3} + \frac{4}{2}\right]\Big|_0^4$$

$$= 2\pi \cdot 5(4)^2\left(-\frac{4}{3} + 2\right) =$$

$$= 2\pi \cdot 80 \cdot (2/3) = \frac{320\pi}{3}$$

Solids of Revolution;



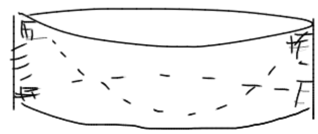
x = radius = x

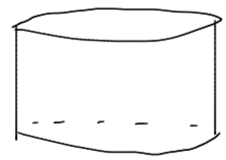
y = height = x^2

Region bound by $y = x^2$

$y = 0$

$x = 4$



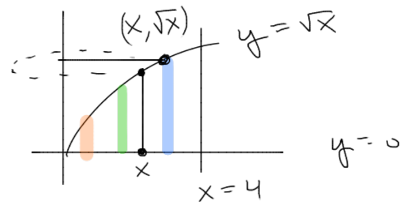


$$V = \int_0^4 2\pi x \cdot f(x)\,dx = 2\pi \int_0^4 \overbrace{x \cdot x^2}^{= x^3}\,dx = 2\pi \cdot \frac{x^4}{4}\Big|_0^4$$

$$= \frac{\pi}{2} \cdot 4^4 = \frac{\pi}{2} \cdot 2^8$$

$$= 2^7 \pi$$

$$128\pi$$

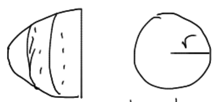


Region :

① Revolve about x-axis : compute vol

② " " y-axis : " "

③ which is larger ?

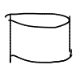
① 

radius touches axis & curve

$r = \sqrt{x}$

$A = \pi r^2 = \pi x$

$$\int_0^4 \pi x \, dx = \pi \frac{x^2}{2} \Big|_0^4 = \frac{16\pi}{2} = 8\pi$$

② Shells  $r = x$, $h = \sqrt{x}$

$$V = 2\pi \int_0^4 x \cdot f(x) = 2\pi \int_0^4 x \cdot \sqrt{x} \, dx = 2\pi \int_0^4 x^{3/2} \, dx$$

$$\frac{2}{5} \cdot 2\pi \cdot x^{5/2} \Big|_0^4 = \frac{4\pi}{5} \cdot x^{5/2} \Big|_0^4 = \frac{4\pi}{5} (\sqrt{x})^5 \Big|_0^4 = \frac{4\pi}{5} (2)^5$$

$$= \frac{4\pi \cdot 32}{5}$$

$4 \cdot 32 = 128 \simeq \frac{125}{5} = 25\pi$

$\approx 25\pi$