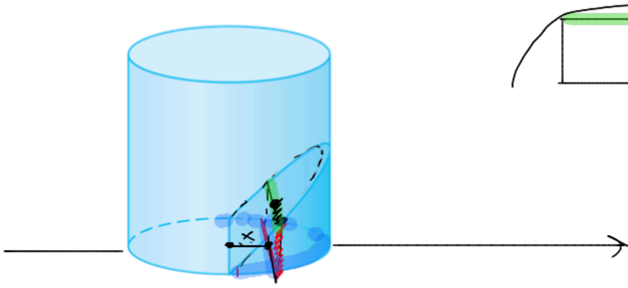
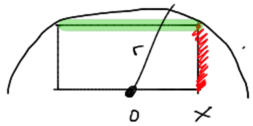
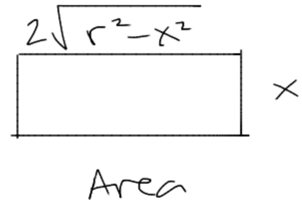
6.2.8

A plane inclined at an angle of 45° passes through a diameter of the base of a cylinder of radius r.





$y = \sqrt{r^2 - x^2}$

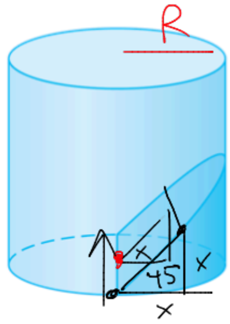


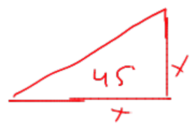
Write an expression for the volume V of the region within the cylinder and below the plane in terms of r.

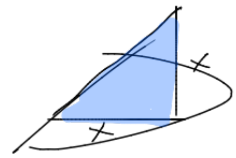
Slice a different way

6.2.8

A plane inclined at an angle of 45° passes through a diameter of the base of a cylinder of radius *r*.







$x^2+y^2=R^2$

solve for x

$x=\sqrt{R^2-y^2}$

$A=\frac{1}{2}\text{base} * \text{height}$

$=\frac{1}{2}x\cdot x=\frac{1}{2}x^2=\frac{1}{2}(R^2-y^2)$

Write an expression for the volume *V* of the region within the cylinder and below the plane in terms of *r*.

$$\int_0^R \frac{1}{2}(R^2-y^2)\,dy$$

Volume: Method of Shells

$$\boxed{V = 2\pi \int_a^b x \cdot f(x)\, dx}$$

b,2,1:

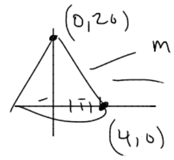
$V = \frac{1}{3}\pi r^2 h$ via

b,2,1,

$h = 20$

$r = 4$

$V = \int_0^{20} 2\pi \cdot x \cdot (-5x+20)$

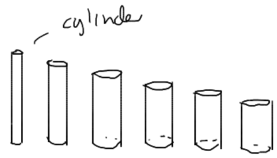


$m = \frac{20-0}{0-4} = -5$

$y = -5x + 20$ — controls height of nested Russian doll cylinder

Shell method

=



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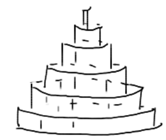
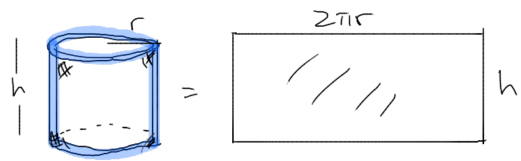


Volume of Cylindrical Shell



$A = 2\pi r h$

$V = 2\pi r h \cdot \Delta r$

$$V = \int_0^4 2\pi \cdot x \cdot (-5x+20) = 2\pi \int_0^4 -5x^2 + 20x = 2\pi\left(-\frac{5x^3}{3} + \frac{20x^2}{2}\right)\Big|_0^4$$

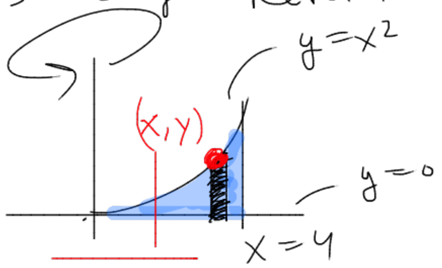
$$= 2\pi \cdot 5x^2\left[-\frac{x}{3} + \frac{4}{2}\right]\Big|_0^4$$

$$= 2\pi \cdot 5(4)^2\left(-\frac{4}{3} + 2\right) =$$

$$= 2\pi \cdot 80 \cdot (2/3) = \frac{320\pi}{3}$$

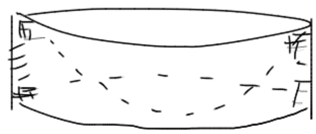
Solids of Revolution;

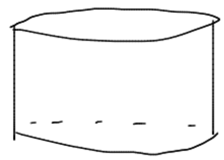
Region bound by $y = x^2$, $y = 0$, $x = 4$



$x = \text{radius} = x$

$y = \text{height} = x^2$



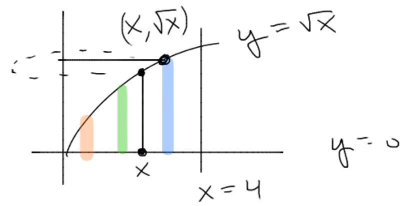


$$V = \int_0^4 2\pi x \cdot f(x)\, dx = 2\pi \int_0^4 \underbrace{x \cdot x^2}_{= x^3}\, dx = 2\pi \cdot \frac{x^4}{4}\Big|_0^4$$

$$= \frac{\pi}{2} \cdot 4^4 = \frac{\pi}{2} \cdot 2^8$$

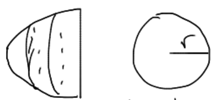
$$= 2^7 \pi$$

$$128\pi$$



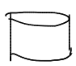
Region :

1. Revolve about x-axis : compute vol
2. " " y-axis : " "
3. which is larger ?

1.  $r = \sqrt{x}$

   $A = \pi r^2 = \pi x$ $\qquad \int_0^4 \pi x \, dx = \pi \frac{x^2}{2} \Big|_0^4 = \frac{16\pi}{2} = 8\pi$

   radius touches axis & curve

2. Shells  $r = x$, $h = \sqrt{x}$

   $V = 2\pi \int_0^4 x \cdot f(x) = 2\pi \int_0^4 x \cdot \sqrt{x} \, dx = 2\pi \int_0^4 x^{3/2} \, dx$

   $\frac{2}{5} \cdot 2\pi \cdot x^{5/2} \Big|_0^4 = \frac{4\pi}{5} \cdot x^{5/2} \Big|_0^4 = \frac{4\pi}{5} \left(\sqrt{x}\right)^5 \Big|_0^4 = \frac{4\pi}{5} (2)^5$

   $4 \cdot 32 = 128 \approx \frac{125}{5} = 25\pi \qquad = \frac{4\pi \cdot 32}{5}$

   $\approx 25\pi$