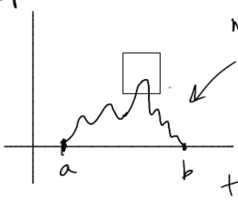
Thur Wk 13

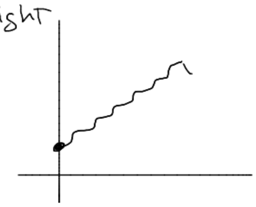
Functions of time, $f(t)$ where $t \in [a, b]$ a closed interval of time

speed

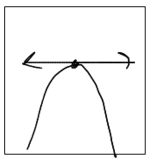


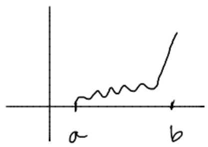
max value occurs @ critical point (1^{st} deriv $= 0$)

height



time



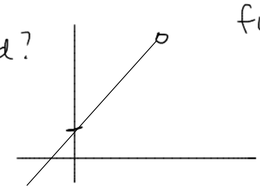


max value occurs @ endpoints

Extreme Value Theorem:

A continuous function on a closed interval achieves its maximum and minimum on that closed interval.

why closed?

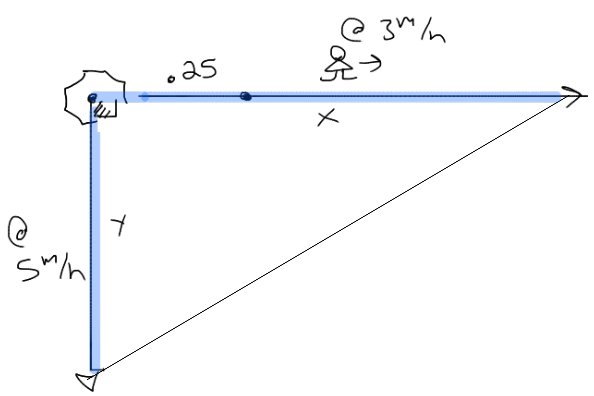


$f(x) = 3x + 1$ on $[0, 4)$

Suppose you are 1/4 mile east of a stop sign, walking @ 3 m/h east (away from the sign).

You friend is at the stop sign, heading south at 5 mi/h.

Produce an equation giving the distances to the stop sign at time t, use this to find how fast the distance between you and your friend is changing 1 hour later.



① visual

② assign variables to given info.

Let $x(t)$ = your dist to sign @ time t.

$$\frac{dx}{dt} = 3$$

Let y = friends dist to sign

$$\frac{dy}{dt} = 5$$

③ Relate variables w/ equation that describes the situation

d = dist. b/w you both

= hypotenuse

$= \sqrt{x^2+y^2}$

④ differentiate to relate $\frac{dx}{dt}$ & $\frac{dy}{dt}$.

$$d(t) = \sqrt{x^2+y^2} = (x^2+y^2)^{\frac{1}{2}}$$

$$d'(t) = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\left(2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}}$$

⑤ $x = \int \frac{dx}{dt} = \int 3\,dt = 3t + C$ b/c of ★ $\underset{.25}{x(0)} = 3(0) + C \Rightarrow C = .25$

$$\boxed{x = 3t + .25}$$

$y = \int \frac{dy}{dt}\,dt = \int 5\,dt = 5t + C$, friend was @ sign so $t=0 \Rightarrow y = 0$ $\underset{0}{y(0)} = 5(0) + C = C$ $\boxed{y = 5t}$

⑥ 1 hour later $x = 3.25$, $y = 5$,

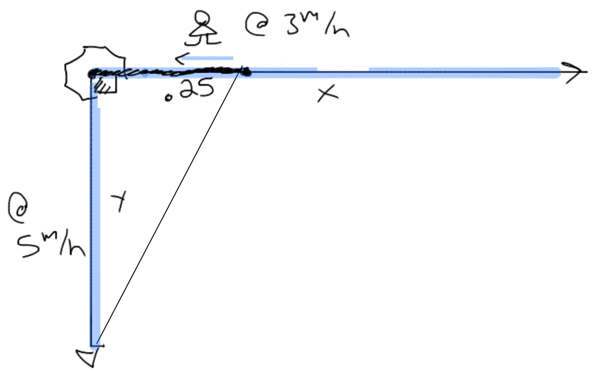
$$d'(1) = \frac{2(3.25)\cdot 3 + 2(5)\cdot 5}{2\sqrt{3.25^2 + 5^2}} \approx 5.8 \ \frac{m}{h}$$

A

Suppose you are 1/4 mile east of a stop sign, walking @ 3 m/h ~~east~~ west (away from the sign).

You friend is at the stop sign, heading south at 5 mi/h.

Produce an equation giving the distances to the stop sign at time t, use this to find how fast the distance between you and your friend is changing 12 min later.



① visual

② assign variables to given info.

Let $x(t)$ = your dist to sign @ time t.

$$\frac{dx}{dt} = -3$$

Let y = friends dist to sign

$$\frac{dy}{dt} = 5$$

③ Relate variables w/ equation that describes the situation

d = dist. b/w you both

= hypotenuse

$= \sqrt{x^2+y^2}$

④ differentiate to relate $\frac{dx}{dt}$ & $\frac{dy}{dt}$.

$$d(t) = \sqrt{x^2+y^2} = (x^2+y^2)^{\frac{1}{2}}$$

$$d'(t) = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\left(2x\cdot\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}}$$

⑤ $x = \int \frac{dx}{dt} = \int -3\,dt = -3t + C$ b/c of A $x(0) = 3(0) + C$, $x(0) = .25 \Rightarrow C = .25$

$$\boxed{x = -3t + .25}$$

$3t = 1/12$

$t = 1/36 \text{ hr} \times \frac{60 \text{ m}}{1 \text{ hr}} = \frac{60}{36} \approx .45$

$y = \int \frac{dy}{dt}\,dt = \int 5\,dt = 5t + C$, friend was @ sign so $t=0 \Rightarrow y = 0$

$y(0) = 5(0) + C = C$, $y(0) = 0$

$$\boxed{y = 5t}$$

12 min ⇒ 1/5 = .2 hour.

⑥ 12 min later $x = -3(.2) + .25$

$= -.6$

$y = 5,$

Find the absolute max / min of

$f(x) = x^4 - 4x^2$ on $[1, 2]$

① Find Local max/min'

$$f'(x) = 4x^3 - 8x = 0$$

set $= 0$

$$4x(x^2 - 2) = 0$$

$$\left.\begin{array}{l} x = 0 \\ x = \pm\sqrt{2} \end{array}\right\} \text{critical pts.}$$

② Compare f @ endpoints w/ f @ critical pts

x	0	$-\sqrt{2}$	$\sqrt{2}$	1	2
$f(x)$	0			-3	0