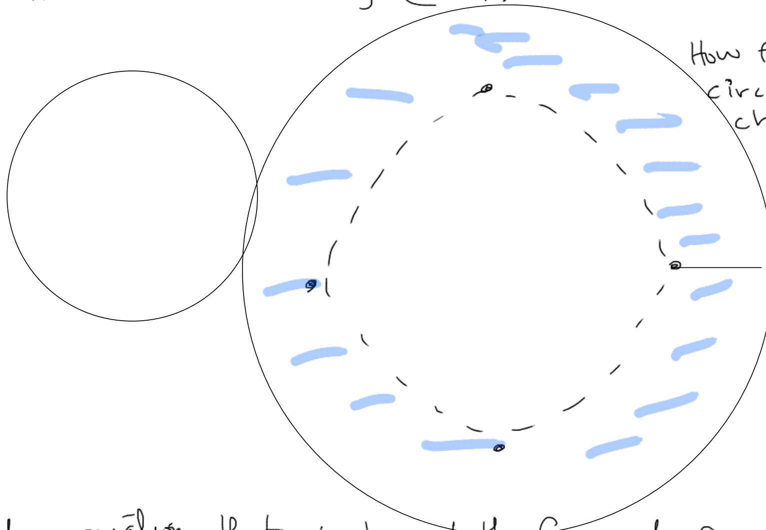
Wed Wk 13

Related Rates:

Imagine the radius of a circle is increasing @ fixed 5 in/s.



How fast is the circumference changing?

$\frac{dC}{dt} = ?$

Given:

r = radius
t = time

$\boxed{\frac{dr}{dt} = 5 \ \frac{in}{s}}$

Goal:

Find: $\frac{dC}{dt}$

How: Relate the Rates: Find equation that involves both C and r. $C_1 = 2\pi r$

do:

take derivative w.r.t. time &

$$C = 2\pi r$$

Note: r = function of time & remember chain rule

$$\boxed{\frac{d}{dt}\left(2\pi(3t+1)^2\right) = 2\pi \cdot 2(3t+1)\cdot 3}$$

$$\frac{dC}{dt} = 2\pi \cdot \underbrace{1 \cdot r^0}_{=1} \cdot \frac{dr}{dt} = 2\pi \frac{dr}{dt}$$

so:

$$\frac{dC}{dt} = 2\pi \cdot 5 = 10\pi$$

In the setting above, how fast is the area changing? when radius is 10. Depends on how big the radius is.

$\frac{dr}{dt} = 5$

$$A = \pi r^2$$

$$\boxed{\frac{dA}{dt} = \frac{d}{dt}\left(\pi r^2\right) = 2\pi r \cdot \frac{dr}{dt}}$$

$$\frac{dA}{dt} = 2\pi(10)\cdot 5 = 100\pi \ \frac{in^2}{s}$$

In the settings above how fast is the area chg when the $\underbrace{\text{diameter}}$ is 8?

still $\frac{dr}{dt} = 5$, $r = ?$ $(\Rightarrow r = 4)$

$2r = D = 8$
$r = 4$

$$\frac{dA}{dt} = 2\pi \cdot 4 \cdot 5 = 40\pi \ \frac{in^2}{s}$$

Spheres

You need to know:

$V = \text{Vol} = \frac{4}{3}\pi r^3$

(note dimension of volume is 3, exponent is 3

$A = $ Surface Area $= 4\pi r^2$

(area is 2-dimensional), exponent is 2.

Note: $\frac{dV}{dr} = \frac{d}{dr}\Big(\underbrace{\frac{4}{3}\pi r^3}_{\text{Vol.}}\Big) = \frac{4}{3}\cdot 3\pi r^2 = \underbrace{4\pi r^2}_{\text{SA.}}$

Suppose a spherical balloon is inflated @ rate of 5 in^3/s.
How fast is the surface changing when $r = 10$ in?

$5\ \frac{in^3}{s}$ = rate of change of vol. $= \boxed{\frac{dV}{dt} \longrightarrow \text{Given}}$

Goal: $\frac{dA}{dt}$ when $r = 10$.

$\overset{\parallel}{\frac{dV}{dt}}$ Relate

$A = 4\pi r^2$

deriv

$\frac{dA}{dt} = \frac{d}{dt}\left(4\pi r^2\right) = 8\pi r\cdot\frac{dr}{dt} \overset{r=10}{=} 8\pi(10)\cdot\frac{dr}{dt}$ stuck until I find $\frac{dr}{dt}$

To find $\frac{dr}{dt}$, use given: $\frac{dV}{dt} = 5 = \frac{4}{3}\cdot 3\pi r^2\cdot\frac{dr}{dt} \longrightarrow 5 = 4\pi\cdot 10^2\cdot\frac{dr}{dt}$

sub $r=10$, solve for $\frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{5}{400\pi}\ \frac{in}{s}}$

Finally, $\frac{dA}{dt} = 80\pi\ \underline{in}\left(\frac{5}{400\pi}\ \frac{in}{s}\right) = 1\ \frac{in^2}{s}$