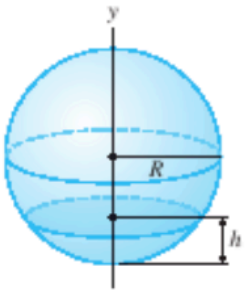
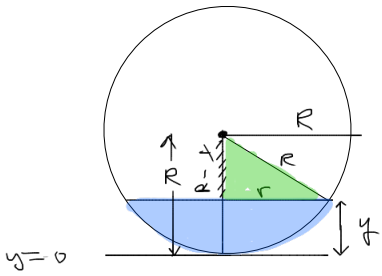
Wed. wk 13

Question 4 of 9

Find the volume of liquid needed to fill a sphere of radius R to height $h = \frac{R}{4}$.





$R^2 = (R-y)^2 + r^2$

get formula for r

$A = \pi r^2$

$V = \int_0^{R/4} \pi (R^2 - (R-y)^2\, dy$

(Use symbolic notation and fractions where needed.)

6-4-4

$y = x^3 + 8$

$y = 10 - x^2$

axis: $x = 2$

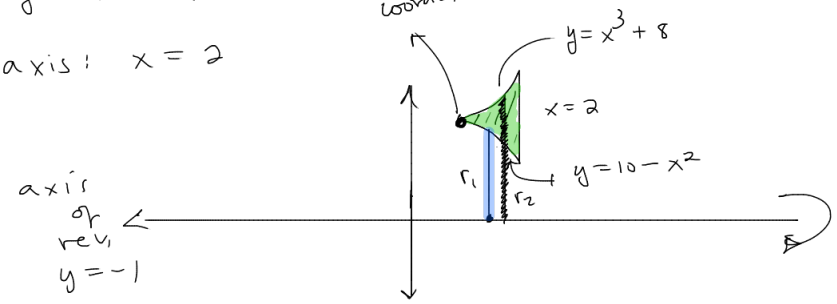
$x^3 + 8 = 10 - x^2$

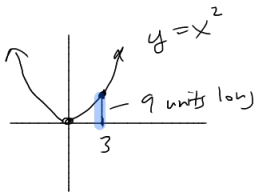
$x^3 + x^2 - 2 = 0$

$x = 1$

set fcns = ?

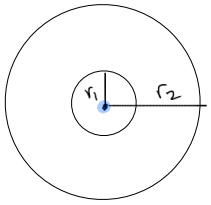
coords?





$V = \int_1^2 \pi \left((x^3+8)^2 - (10 - x^2)^2 \right) dx$

washer!



radius: segments touch axis & curve

$r_1 = 10 - x^2$

$r_2 = x^3 + 8$

$A = \pi r_2^2 - \pi r_1^2$

$A = \pi \left((x^3+8)^2 - (10 - x^2)^2 \right)$

Applications

- optimization
- related rates

differential

integral

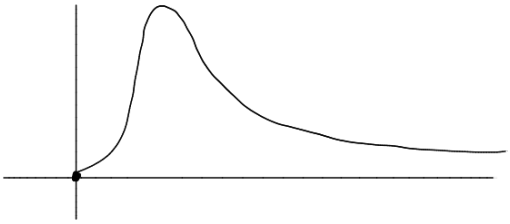
- volume
- area
- work

Question 3 of 10

The concentration of a drug in the bloodstream t hours after injection into the body is given by the function C.

$$C(t) = \frac{4t}{0.9 + t^2}$$

When is the concentration of a drug in the bloodstream the greatest? Round your answer to two decimal places.

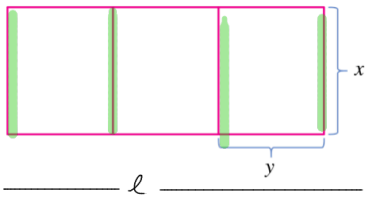


All optimization problems require:

1. Take derivative
2. set = 0, solve

Hint: quotient rule

Your task is to design a rectangular industrial warehouse consisting of three separate spaces of equal size as in the figure.



The wall materials cost $500 per linear meter and your company allocates $7,200,000 for that part of the project involving the walls.

Which dimensions maximize the area of the warehouse?

Goal:
dims: length = $3y$
width = x
that maximize area
(1) get area function
(2) take deriv, set = 0, solve

$A = \ell \cdot w = 3yx$ — two variables! need to get down to 1

key: "constraint" ← parse sentence ★ ← relation b/w x & y

Total length: $4x + 6y$

Total Cost: $500(4x + 6y) = 7{,}200{,}000$

isolate y: $4x + 6y = \frac{7200000}{500} = 14{,}500$

$$y = \frac{14500 - 4x}{6}$$

objective function

$$A = 3\left(\frac{14500 - 4x}{6}\right)x = \frac{1}{2}(14500 - 4x)x = (7250 - 2x)x = 7250x - 2x^2$$

$$A' = 7250 - 4x = 0 \Rightarrow x = \frac{7250}{2.2} = \frac{3625}{2} = 1812.5 \text{ m} = \text{width}$$

$$\text{length} = 3y = 3\left(\frac{14500 - 4(1812.5)}{6}\right)$$