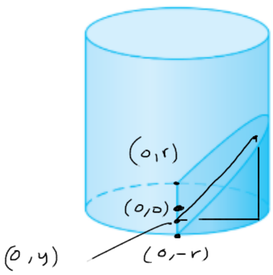
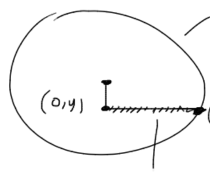
Wk 14 – Mon

- Exam 4 tomorrow
- Final ; Monday 12 pm.

A plane inclined at an angle of 45° passes through a diameter of the base of a cylinder of radius r.

$A = \frac{1}{2} b \cdot h = \frac{1}{2}(r^2 - x^2)$

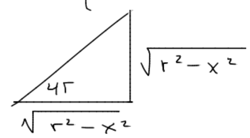




$x^2 + y^2 = r^2$

$(\sqrt{r^2 - y^2}, y)$

$(x, y) = (x, \sqrt{r^2 - x^2})$



length = difference b/w x-coords of endpoints

$= \sqrt{r^2 - y^2}$

Write an expression for the volume V of the region within the cylinder and below the plane in terms of r.

$$\int_{\min y}^{\max y} A(y)\,dy = \int_{-r}^{r} \frac{1}{2}(r^2 - y^2)\,dy$$

1. Evaluate the limit . . . . (a)

$$\lim_{x\to 0^+} \frac{2x^4 - 8x^2}{x^4 + 4x^2} = \frac{0}{0} \quad \text{L'H}$$

$$\lim_{x\to 0^+} \frac{8x^3 - 16x}{4x^3 + 8x} = \frac{0}{0}$$

$$\text{L'H} \quad \lim_{x\to 0^+} \frac{24x^2 - 16}{12x^2 + 8} = -2$$

(b)

$$\lim_{x\to 0} \frac{\sin x - x}{x^3} = \frac{\sin 0 - 0}{0^3} = \frac{0}{0}$$

L'H

$$\lim_{x\to 0} \frac{\cos x - 1}{3x^2} = \frac{\cos 0 - 1}{3\cdot 0^2} = \frac{0}{0}$$

$$\text{L'H} \quad = \lim_{x\to 0} \frac{-\sin x}{6x} = \frac{0}{0}$$

$$\text{L'H} \quad \lim_{x\to 0} = \frac{-\cos x}{6} = \frac{-1}{6}$$

curveball

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = \left(1+\frac{1}{\infty}\right)^\infty = 1^\infty \quad ???$$

1 ||

set y $\Rightarrow$ $\ln(y) = \ln\left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right) = \lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x$

$$= \lim_{x\to\infty} x\cdot\ln\left(1+\frac{1}{x}\right) = \infty\cdot\overbrace{\ln(1+0)}^{=0} = \infty\cdot 0 \quad ???$$

$$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \frac{\ln 1}{\frac{1}{\infty}} = \frac{0}{0} \quad \text{L'H}$$

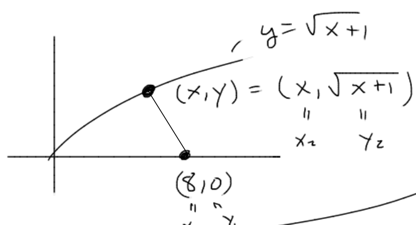
$$= \lim_{x\to\infty} \frac{\frac{1}{1+1/x}\left(-\frac{1}{x^2}\right)}{-1/x^2} = \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1 = \ln(y)$$

$y = e$

2. Find the point on the ~~line~~ curve $y = \sqrt{x+1}$ that is closest to the point $(8, 0)$.

optimization: 1. get model (function) 2. take deriv., set $= 0$, solve

function = distance



$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} = \sqrt{(8-x)^2 + (0-\sqrt{x+1})^2}$$

$$= \sqrt{(8-x)^2 + (x+1)}$$

goal find where min distance occurs, this is equiv. to finding when min $(\text{distance})^2$ occurs.

$$\text{Min } D(x) = (8-x)^2 + (x+1)^2$$

$$D'(x) = 2(8-x)(-1) + 1$$

$$= -16 + 2x + 1 = 0$$

$2x = 15$ $y = \sqrt{8.5}$

$x = 7.5$ $y = \sqrt{7.5+1}$

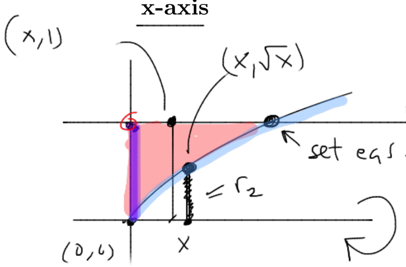
$(7.5, \sqrt{8.5})$

Note ↙ — vert

3. Consider the region bound by $y = 1$, $x = 0$ and $y = \sqrt{x}$. Find the volume of the solid of revolution when the region is revolved about the:

horizontal

x-axis



washer:
- integrate along axis (wrt x)
- radii touch axis & curve

set eqs = $\sqrt{x} = 1$

$\Rightarrow x = 1$

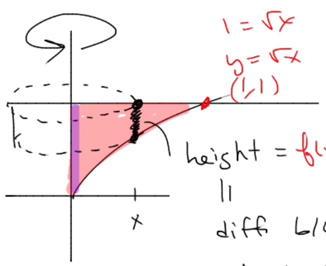
r_2 = length of [mark] = distance b/w y-values

$r_2 = \sqrt{x}$

$r_1 = 1$

$$\int_{\min x}^{\max x} \pi\left(r_1^2 - r_2^2\right) dx = \pi\int_0^1 (1 - x)\, dx = \pi\left(x - \frac{x^2}{2}\right)\Big|_0^1 = \pi\left(1 - \frac{1}{2}\right) = \frac{\pi}{2}$$

4. Revolve the region above about the **y-axis** and compute the volume of the resulting solid.



height = f(x)
||
diff b/w y-coords of endpts

Shell: $V = \int_{\min x}^{\max x} 2\pi x \cdot f(x)\, dx$

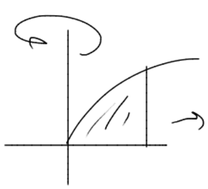
height

$$= \int_0^1 2\pi (x)\left(1 - \sqrt{x}\right) dx$$

$$= 2\pi \int_0^1 x - x^{3/2}\, dx = 2\pi\left(\frac{x^2}{2} - 2\frac{x^{5/2}}{5}\right)\Big|_0^1$$

$$= 2\pi\left(\frac{1}{2} - \frac{2}{5}\right) = 2\pi\left(\frac{1}{10}\right)$$

$$= \frac{\pi}{5}$$



$\frac{4\pi}{5}$

π