Wk 14 – Mon

Exam 4 – tomorrow

Final : thursday 10 am

1. Evaluate the limit . . . . (a)

$$\lim_{x\to 0^+} \frac{2x^4 - 8x^2}{x^4 + 4x^2} = \frac{0}{0}$$

$$\frac{\cancel{x^2}(2x^2 - 8)}{\cancel{x^2}(x^2+4)}$$

$$\lim_{x\to 0} \frac{2x^2-8}{x^2+4} = -2$$

L'H : $\lim_{x\to 0^+} \frac{8x^3 - 16x}{4x^3 + 8x} = \frac{0}{0}$

L'H $\lim_{x\to 0^+} \frac{24x^2 - 16}{12x^2 + 8} = -2$

(b)

$$\lim_{x\to 0} \frac{\sin x - x}{x^3} = \frac{\sin 0 - 0}{0^3} = \frac{0}{0}$$

L'H $\lim_{x\to 0} \frac{\cos x - 1}{3x^2} = \frac{1-1}{0} = \frac{0}{0}$

L'H $\lim_{x\to 0} \frac{-\sin x}{6x} = \frac{0}{0}$

L'H $\lim_{x\to 0} \frac{-\cos x}{6} = -\frac{1}{6}$

curve-ball:

$\lim_{x\to +\infty} \left(1 + \frac{1}{x}\right)^x = (1+0)^\infty = 1^\infty$ No L'H

① $= y$

② $\ln(y) = \ln\left(\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x\right)$ no L'H

$= \lim_{x\to+\infty} \ln\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} x \cdot \ln\left(1+\frac{1}{x}\right)$ $\infty \cdot 0$

③ $\lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \frac{\ln(1)}{0} = \frac{0}{0}$ L'H

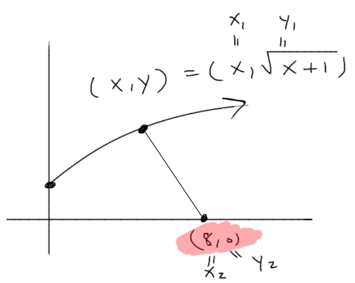
④ $\lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$

⑤ $= \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1$

⑥ $\ln(y) = 1$

$y = e$

2. Find the point on the ~~line~~ curve $y = \sqrt{x+1}$ that is closest to the point $(8, 0)$.



optimization ⇓ create a function that models distance, then take deriv, set $= 0$

Dist. Formula: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$D(x) = \sqrt{(x-8)^2 + (\sqrt{x+1} - 0)^2}$

$= \sqrt{(x-8)^2 + \sqrt{x+1}}$

$D^2 = (x-8)^2 + x + 1$

$D'(x) = \frac{1}{2}\left((x-8)^2 + x + 1\right)^{-1/2} \cdot (2(x-8) + 1) = 0$

$= \frac{2x - 16 + 1}{2\sqrt{(x-8)^2 + x + 1}} = 0 \iff 2x - 16 + 1 = 0$

$2x = 15$

$x = 7.5$

$y = \sqrt{8.5}$

$(7.5, \sqrt{8.5})$

When minimizing $\sqrt{f(x)}$, just minimize $f(x)$

and

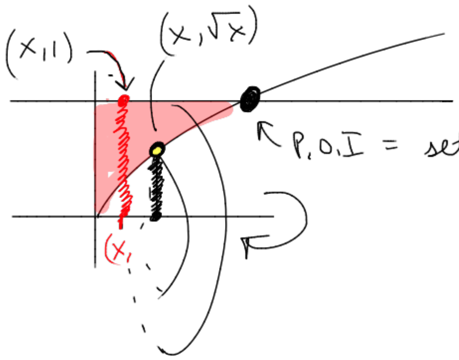
when minimizing $(f(x))^2$ just minimize $f(x)$

3. Consider the region bound by $y = 1$, $x = 0$ and $y = \sqrt{x}$. Find the volume of the solid of revolution when the region is revolved about the:

washer:

- Int. along axis (wrt x)
-

x-axis



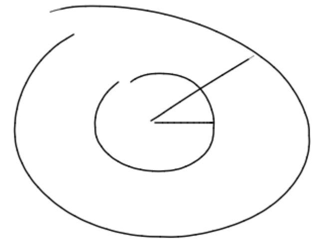
$y = \sqrt{x}$

$y = 1$

P.O.I = set eq's equal

$\sqrt{x} = 1$

$x = 1$



radii touch axis & curve

length of  = diff. in y-coords: $\sqrt{x}$

$r_2 = \sqrt{x}$

$r_1 = 1$

$A(x) = \pi(r_1^2 - r_2^2)$

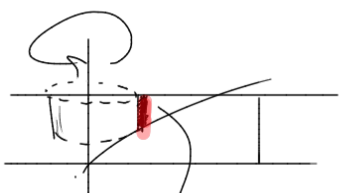
$= \pi(1 - x)$

$\star$

$$\int_0^1 \pi(1-x)\,dx = \pi\left(x - \frac{x^2}{2}\right)\Big|_0^1$$

$$= \pi\left(1 - \tfrac{1}{2}\right) = \frac{\pi}{2}$$

4. Revolve the region above about the **y-axis** and compute the volume of the resulting solid.



$y = 1$

length: $1 - \sqrt{x}$

Shell:

$$V = \int_{\min x}^{\max x} 2\pi x \cdot f(x)\,dx$$

$$V = \int_0^1 2\pi x \cdot \underline{(1-\sqrt{x})}\,dx$$

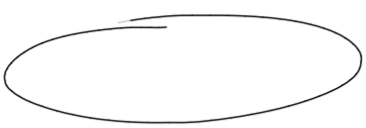
$$= 2\pi \int_0^1 x - x^{3/2}\,dx$$

$$= 2\pi\left(\frac{x^2}{2} - \frac{2x^{5/2}}{5}\right)\Big|_0^1$$

$$= 2\pi\left(\tfrac{1}{2} - \tfrac{2}{5}\right) = 2\pi\left(\tfrac{1}{10}\right)$$

$$= \frac{\pi}{5}$$

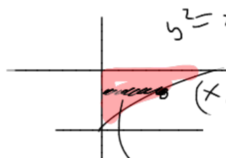
disc:



$r_1 = y^2$

$y = \sqrt{x}$

$y^2 = x$



$(x, \sqrt{x})$

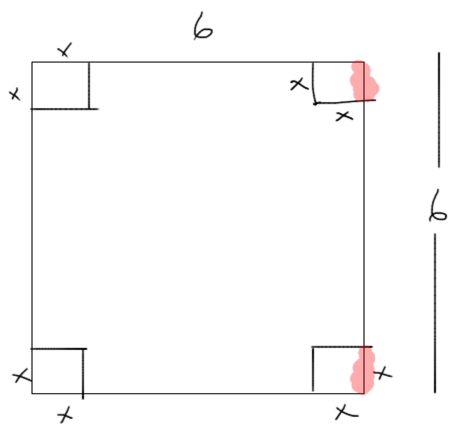
(y^2, y)

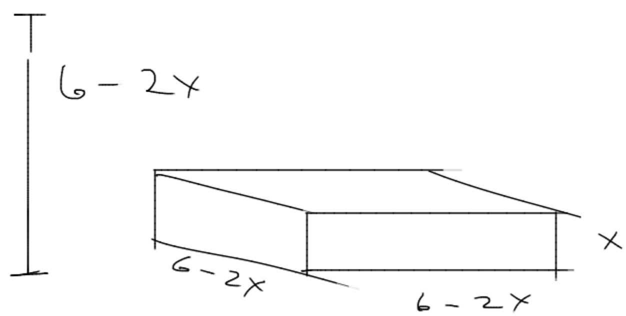
length = diff in $\underline{\underline{x}}$-coords

$$\int_0^1 \pi r_1^2\,dy = \int_0^1 \pi (y^2)^2\,dy = \pi\int_0^1 y^4 = \pi \frac{y^5}{5}\Big|_0^1 = \frac{\pi}{5}$$

9. A fence 7 feet tall runs parallel to a tall building at a distance of 6 feet from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

10. A box with an open top is to be constructed from a square piece of cardboard, 6 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.





$$V = l \cdot w \cdot h = x(6-2x)^2$$

$$V' = 1 \cdot (6-2x)^2 + x(2(6-2x)(-2))$$

$$= (6-2x)\left[\underbrace{(6-2x) - 4x}_{6-6x}\right] = 0$$

$6 - 2x = 0 \implies x = 3$ — too big of a cut!

$6 - 6x = 0 \quad x = 1 \implies V(1) = 1 \cdot (6-2)^2$

$= 16 \text{ ft}^3$