L'Hospital's Rule.

If $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$ then $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to c} \dfrac{f'(x)}{g'(x)}$

Ex $$\lim_{x \to \infty} \frac{x^2 + 100x}{x^2 + 1} = \frac{\overset{\infty}{\overset{\parallel}{\infty^2}} + \overbrace{100 \cdot \infty}^{=\infty}}{\underbrace{\infty^2 + 1}_{\infty}} = \frac{\infty + \infty}{\infty} = \frac{\infty}{\infty}$$

Note:

$\infty - \infty \neq 0$

1^{∞} = undefined

L'Hospital's Rule applies:

$$\lim_{x \to \infty} \frac{2x + 100}{2x} = \frac{\infty}{\infty}$$ L'H again

$$\lim_{x \to \infty} \frac{2}{2} = \lim_{x \to \infty} 1 = 1$$

Ex $$\lim_{x \to 4} \frac{x - 4}{x^2 - 16} = \frac{0}{0}$$ L'H applies

$$\lim_{x \to 4} \frac{1}{2x} = \frac{1}{8}$$

Ex $$\lim_{x \to \infty} \frac{5x^3 + x + 100}{3x^3 + 1} = \frac{5}{3}$$

$\overline{\text{L'H}}$

$$\lim_{x\to 0} \frac{\sin x - x}{x^3} \overset{\text{direct sub}}{=} \frac{\sin 0 - 0}{0^3} = \frac{0}{0} \qquad L'H$$

$$\lim_{x\to 0} \frac{\cos x - 1}{3x^2} = \frac{1-1}{0} = \frac{0}{0} \qquad L'H$$

$$\lim_{x\to 0} \frac{-\sin x}{6x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{-\cos x}{6} = -\frac{1}{6}$$

8. Evaluate the indefinite integral.

(a) $\int \sin^2(x)\cos(x)dx =$

(b) $\int e^{3x-2}dx =$

(c) $\int \frac{2x}{x^2+1}dx =$

(d) $\int \frac{7+2x}{x^2+1}dx =$

$$= \boxed{\left(\frac{x^4+1}{6}\right)^6 + C}$$

(e) $\int 4x^3(x^4+1)^5 dx =$ $\int \cancel{4x^3}(u)^5 \frac{1}{\cancel{4x^3}}du = \int u^5 du = \frac{u^6}{6} + C$

$\boxed{u = x^4 + 1}$

$\frac{du}{dx} = 4x^3 \Rightarrow du = 4x^3\,dx \Rightarrow \boxed{\frac{1}{4x^3}du = dx}$

(f) $\int \frac{2x^3}{\sqrt{x^4+5}}dx =$