Grade:

$$.25(\text{WEBWORK}) + .25(\text{Final}) + .50\left(\text{Exam}_{\text{Avg}}\right)$$

$$.25(100) + .75\left(\text{Exam}_{\text{Avg}}\right) = \underline{\qquad\qquad}$$

Many similar problems appear on Final.

Name: ______________________________________ MA161 · Exam 4 · April 22, 2024

$x_2 = x,\ y_2 = \sqrt{x_2+1}$ $\quad x_1 = y_1$

1. Find the point on the line $y = \sqrt{x+1}$ that is closest to the point $(8, 0)$.

optimization
closest $\Rightarrow$ minimize distance formula: $d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$d = \sqrt{(8-x)^2 + (0-\sqrt{x+1})^2} = \sqrt{(8-x)^2 + (x+1)} = \sqrt{64 - 16x + x^2 + x + 1} = \sqrt{x^2 - 15x + 65}$

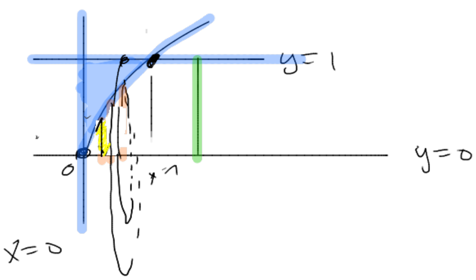
equivalently, find where min of d^2 occurs:

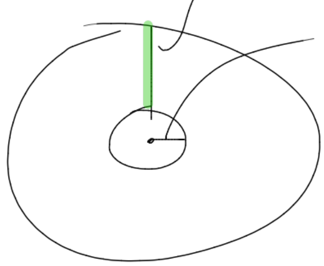
$D = d^2 = x^2 - 15x + 65$

$D' = 2x - 15 = 0 \Rightarrow x = 15/2 = 7.5 \Rightarrow y = \sqrt{7.5+1} = \sqrt{8.5}$ $\quad \boxed{(7.5, \sqrt{8.5})}$

2. Consider the region bound by $y = 1$, $x = 0$ and $y = \sqrt{x}$. Find the volume of the solid of revolution when the region is revolved about the:

x-axis





$\pi \int_0^1 r_o^2 - r_i^2 \, dx$

$\pi \int_0^1 1 - \sqrt{x}^2 \, dx = \pi \int_0^1 1 - x \, dx$

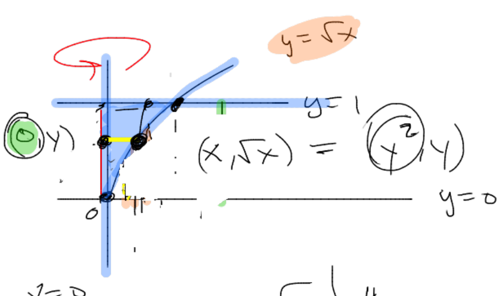
$\pi \left(x - \frac{x^2}{2} \Big|_0^1 \right) = \frac{\pi}{2}$

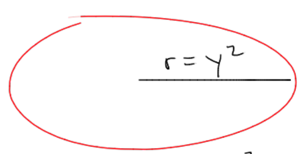
3. Revolve the region above about the **y-axis** and compute the volume of the resulting solid.

change var to y:

$y = \sqrt{x} \longrightarrow y^2 = x$

DISC





$A = \pi r^2 = \pi (y^2)^2$

$\pi \int_0^1 y^4 \, dy = \pi \frac{y^5}{5} \Big|_0^1 = \frac{\pi}{5}$

4. Find the equation of the tangent line to the graph of $y = \overset{\text{product}}{(x^2 + 1)\sin x}$ at $x = 0$.

point: $x=0,\ y=(0^2+1)\cdot\sin(0) = 0 \qquad (0,0)$

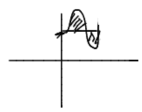
slope = derivative @ given x-coord

$f'(x) = 2x\cdot\sin x + (x^2+1)\cos(x)$

$f'(0) = \underbrace{2\cdot 0\cdot\sin 0}_{0} + \underbrace{(0^2+1)\cos(0)}_{1} = 1$

$y - y_1 = m(x - x_1)$

$\boxed{y = x}$



5. Find the average value of the function below from $x = 0$ to $x = 2\pi$.

$$f(x) = \sin(3x) + 1$$

$\frac{1}{b-a}\int_a^b f(x)\,dx$

$\frac{1}{2\pi}\int_0^{2\pi} \sin(3x)+1\,dx = \frac{1}{2\pi}\left[\int_0^{2\pi} \underbrace{\sin(3x)}_{u=3x} + \underbrace{\int 1\,dx}_{x}\right]$

$du = 3dx$

$\frac{1}{3}du = dx$

$= \frac{1}{2\pi}\left[\frac{1}{3}\int_0^{2\pi} \sin(u)\,du + x\Big|_0^{2\pi}\right]$

$= \frac{1}{2\pi}\left[\underbrace{-\frac{1}{3}\cos(3x)\Big|_0^{2\pi}}_{=0} + \underbrace{x\Big|_0^{2\pi}}_{2\pi}\right] = 1$

6. Suppose the volume of a spherical balloon increases at a rate of $24\frac{cm^3}{sec}$. Find the rate that its diameter is increasing when the diameter is $3cm$.

$\frac{dV}{dt} = 24$, $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{4}{3}\pi\frac{D^3}{8} = \frac{\pi}{3}D^3$

$\frac{dV}{dt} = \frac{\pi}{6}\cdot 3\cdot D^2\cdot\frac{dD}{dt}$

set $D = 3$, sub $\frac{dV}{dt} = 24$, solve for $\frac{dD}{dt}$

$\frac{24}{\frac{\pi}{6}\cdot 3\cdot(3)^2} \approx 1.5\,\frac{cm}{s} = \frac{dD}{dt}$

7. Find the absolute maximum and absolute minimum of the function on the indicated interval.

optimization

take deriv., set = 0, find c.P.'s

$$f(x) = \frac{x^4}{4} - 2x^2 + 1, \; [-3, 1]$$

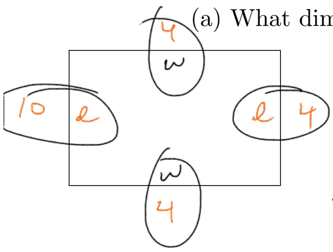
make table, compare endpoints & c.P.'s

value of f @

$lw = 28, \; l = 28/w$

8. A gardener is planning to build a rectangular fence which encloses 28 ft^2. One of the sides is to be made of stone which costs $10 \frac{\$}{ft}$, and the remaining sides are to be made of wood which costs $4 \frac{\$}{ft}$.

(a) What dimensions minimize the cost of such a fence?



$C = 14l + 8w$

$C = 14(28/w) + 8w$

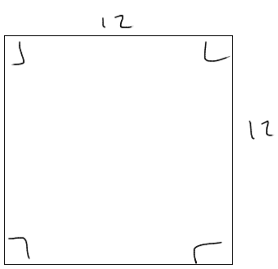
Take deriv. set = 0, solve

this is width that gives least cost

(b) What is the mimimum cost?

Plug ans to (a) into C.

9. A box with an open top is to be constructed from a square piece of cardboard, 12 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.



$V = x(12-2x)^2 \rightarrow 1(12-2x)^2 + x(2(12-2x)(-2))$

take deriv set = 0, solve

$V' = [12-2x][(12-2x) - 4 \cdot x] = 0$

$12 - 2x = 0$	$12 - 6x = 0$
$x = 6$	$x = 2$ ✓
too much	

$V(2) = 2(12 - 2(2))^2 =$