wk 14 -Wed

Exam 4 grades: posted

Educat:

I Estimated Grade: $.75(\text{Exam Avg}) + .25(\text{Homework Avg})$

II Estimated_Grade_Drop: $.75(\text{Exam Avg w/ lowest score dropped}) + .25(\text{Homework Avg})$

You'll receive this if: Final Exam Score > Exam Avg. (w/ lowest score dropped)

Math 161 - Calculus - Exam 4 **Name:** ____________________

December 3, 2024

Show your work to receive full credit.

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1. Use L'Hôpital's Rule to evaluate the limits below.

(1.1)

$$\lim_{x\to\infty} \frac{x^3+1000}{x+e^x} \xrightarrow{L'H} \frac{3x^2}{1+e^x} \xrightarrow{L'H} \frac{6x}{e^x} \xrightarrow{L'H} \frac{6}{e^x} \to 0$$

(1.2) Remember this limit from earlier in the semester?

$$\lim_{x\to 0} \frac{\sin x}{x} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$

(1.3)

$$y \overset{set}{=} \lim_{x\to 0} (1-3x)^{1/x} = 1^{\infty} \; ?$$

$$\boxed{\frac{1}{x}\cdot A = \frac{A}{x}}$$

$$\ln(y) = \ln(\lim(\cdots))$$

$$= \lim\left(\frac{1}{x}\ln(1-3x)\right)$$

$$= \lim_{x\to 0} \frac{\ln(1-3x)}{x} = \frac{\ln 1}{0} = \frac{0}{0}$$

$$L'H \quad = \lim \frac{\frac{1}{1-3x}\cdot(-3)}{1} = \frac{\frac{1}{1-0}(-3)}{1} = -3$$

$$y = e^{-3}$$

likely on Final

2. Find the point on the curve $y = x^2$ that is closest to the point $(3, \frac{1}{2})$.

⇒ minimize distance

① get formula

② take deriv, set = 0, solve

Hint: trust chain rule, instead of expanding first

distance

$x_1 = x$ | $x_2 = 3$

$y_1 = x^2$ | $y_2 = \frac{1}{2}$

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

since $D > 0$, D is a minimum whenever D^2 is.

$$D^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x-3)^2 + (x^2 - \tfrac{1}{2})^2$$

$$(D^2)' = 2(x-3)\cdot 1 + \overset{4x}{2}(x^2 - \tfrac{1}{2})\cdot 2x$$

$$= 2x - 6 + 4x^3 - 2x = -6 + 4x^3 = 0$$

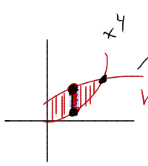
$$x = \sqrt[3]{6/4} \approx 1.1$$

$$y = (1.1)^2$$

$(1.1, 1.1)$

3. Find the **area** bounded by the graphs of $y = x^4$ and $y = \sqrt{x} + 254$ and $x = 0$.

$$A = \int_a^b \text{height}\, dx = \int_0^b \sqrt{x} + 254 - x^4\, dx$$



height = top y − bottom y

bounds

$x^4 = \sqrt{x} + 254$

$x^4 - 254 = \sqrt{x}$

$(x^4 - 254)^2 = x$

↓

$x^8 - 508x^4 + 254^2 = x$

stuck

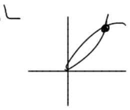
① $x = 1$?

$x = 2$?

$x = 3$

$x = 4$ ✓ $4^4 = 2^8 = 256 = \sqrt{4} + 254$

② graph



$$\frac{2}{3}x^{\frac{3}{2}} + 254x - \frac{x^5}{5}\Big|_0^4 =$$

$$x\left(\frac{2}{3}x^{\frac{1}{2}} + 254 - \frac{x^4}{5}\right)\Big|_0^4 = 4\left(\frac{2}{3}\cdot 2 + \overbrace{254 - \frac{256}{5}}^{250}\right) \approx 880$$

$\frac{250}{5}$ 50

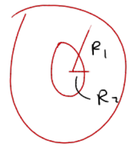
4. Consider the region bound by $x = 0$, $y = 5$ and $y = \sqrt{x} + 1$. Find the volume of the solid of revolution when the region is revolved about the:

① Hole in solid b/c area isn't attached along edge to axis

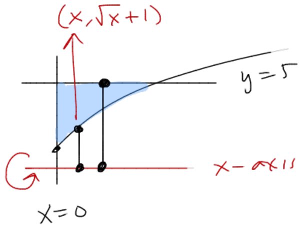
⇒ washer

slice ⊥ axis

vertical



x-axis



$\sqrt{x}+1 = 5 \Rightarrow \sqrt{x} = 4$, $x = 16$

② R_1 : touch axis & top curve $= 5$

R_2 touch axis & bottom curve $= \sqrt{x} + 1$

③ $A(x) = \pi(R_1^2 - R_2^2) = \pi(25 - (\sqrt{x}+1)^2)$

④ $V = \int_0^{b=16} \pi(25 - (\sqrt{x}+1)^2)\,dx = \pi\int_0^{16} 24 - x - 2x^{\frac{1}{2}}\,dx = \pi\left[24x - \frac{x^2}{2} - \frac{4x^{3/2}}{3}\right]\Big|_0^{16}$

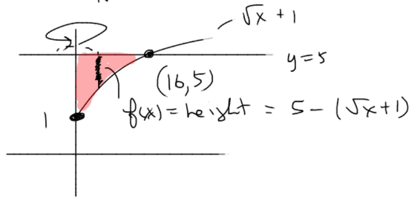
algebra: FOIL

⑤ $\pi x\left[24 - \frac{x}{2} - \frac{4x^{1/2}}{3}\right]_0^{16} = \pi\cdot 16\left[24 - 8 - \frac{4}{3}\cdot 4\right] = \pi 16[10.7] \approx 160\pi$

$16/3 = 5.\bar{3}$

5. Revolve the region above about the **y-axis** and compute the volume of the resulting solid.

No Holes: disc / shell ⇒ int ⊥ axis wrt x



$V = \int_0^{16} 2\pi x\, f(x)\,dx$

$= 2\pi\int_0^{16} x\cdot(4 - \sqrt{x})\,dx$

$= 2\pi\int_0^{16} 4x - x^{3/2}\,dx$

Shell

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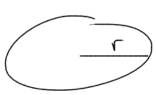
disc int. ALONG axis ⇒ wrt y!

$y = \sqrt{x}+1 \Rightarrow y - 1 = \sqrt{x}$ $\boxed{(y-1)^2 = x}$



$r = x\text{-coord} = (y-1)^2$

$\int_{\min y}^{\max y} \pi r^2\,dy$, $\max y = 5$, $\min y = 1$

u-sub $u = y - 1$, $du = dy$ | $y = 1 \Rightarrow u = 0$, $y = 5 \Rightarrow u = 4$

$\int_1^5 \pi((y-1)^4)\,dy = \int_0^4 \pi u^4\,du = \frac{\pi u^5}{5}\Big|_0^4 = \frac{1024\pi}{5}$

6. A box with an open top is to be constructed from a square piece of cardboard, 12 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

$V = x(12-2x)^2$

$V' = 1(12-2x)^2 + x(2(12-2x)^1(-2)) = (12-2x)\left[\overbrace{(12-2x)-4x}^{(12-6x)}\right] = 0$

product
chain

$12-2x=0 \quad x=6$

$12-6x=0 \quad x=2$

$V(2) = 2(12-2\cdot 2)^2$

$= 128$

7. What are the dimensions of biggest rectangular fence (in area) you can build if you can only spend \$140 and one of the sides is to be made of stone which costs $10\frac{\$}{ft}$, and the remaining sides are to be made of wood which costs $4\frac{\$}{ft}$?

$A = l w$

maximize A

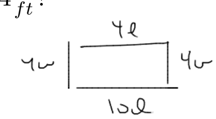
$\Rightarrow$ solve $A' = 0$

use chain product rule

$140 = 14l + 8w$

$\frac{140-14l}{8} = w$

sub



5

8. Suppose each edge of a cube increases at a rate of $4 \frac{in}{sec}$.

(8.1) How fast is the volume growing at the instant the edge has length 5 in?

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} = 4$$

vol increase rate

(8.2) How fast is the volume growing at the instant the edge has length 10 in?