Wk 14 – Wed

EducaT ———————————————

open!

- grades posted

I. est. grade = .75 (exam avg) + .25 (homework)

II. est_grade_dropped = .75 (ex. avg w/ lowest dropped) + .25 (homework)

III if your final exam grade ≥ exam avg. w/ dropped

Math 161 - Calculus - Exam 4 **Name:** ____________________

December 3, 2024

Show your work to receive full credit.

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1. Use L'Hôpital's Rule to evaluate the limits below.

(1.1)

$$\lim_{x\to\infty} \frac{x^3+1000}{x+e^x} \longrightarrow$$

L'H 3x

$\frac{3x^2}{1+e^x} \to \frac{6x}{e^x} \to \frac{6}{e^x} \to \emptyset$ as $x\to\infty$

(1.2) Remember this limit from earlier in the semester?

$$\lim_{x\to 0} \frac{\sin x}{x} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$

(1.3)

$$y = \lim_{x\to 0} (1-3x)^{1/x}$$

$\ln(y) = \ln\left(\lim (1-3x)^{\frac{1}{x}}\right)$

$= \lim_{x\to 0}\left(\frac{1}{x}\ln(1-3x)\right)$

$= \lim_{x\to 0} \frac{\ln(1-3x)}{x} = \frac{0}{0}$

$= \lim_{x\to 0} \frac{\frac{1}{1-3x}\cdot -3}{1} = -3$

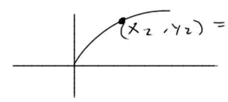
$y = e^{-3}$

good Find Q

2. Find the point on the curve $y = x^2$ that is closest to the point $(3, \frac{1}{2})$.

x_1 y_1

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



$D' = 0$ or equiv.

$$D = \sqrt{(3-x)^2 + (\tfrac{1}{2} - x^2)^2}$$

D is a min whenever D^2 is (since its positive)

minimize D^2

$(D^2) = (3-x)^2 + (\frac{1}{2} - x^2)^2$ no need to expand (trust your chain rule)

$-4x(\frac{1}{2} - x^2)$

$(D^2)' = -2(3-x) + 2(\frac{1}{2} - x^2)(-2x)$

$= -6 + \underbrace{2x - 2x}_{=0} + 4x^3 = 0$

$(1.1, 1.1)$

$4x^3 = 6$ $x = \sqrt[3]{\frac{6}{4}} \approx 1.1 \Rightarrow y = (\sqrt[3]{\frac{6}{4}})^2 \approx 1.1$

$256 = 2^8$

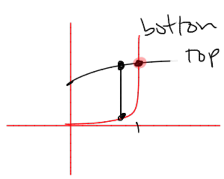
3. Find the **area** bounded by the graphs of $y = x^4$ and $y = \sqrt{x} + 254$ and $x = 0$.

$\int_a^b$ height dx

touch curves

top − bottom

$(\sqrt{x} + 254 - x^4)$



$$\int_0^b \sqrt{x} + 254 - x^4 \, dx$$

$\sqrt{x} + 254 = x^4$

$\sqrt{x} = x^4 - 254$

$x = (x^4 - 254)^2$

$= (x^8 - 2x^4(254) + (254)^2)$

stuck

① w/o calculator:

$x = 1$

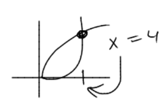
$x = 2$

$x = 3$

$x = 4$

$\sqrt{4} + 254 = 4^4$, $\sqrt{4} = 2$, $4^4 = 2^8$

② graph:



$$\int_0^4 \sqrt{x} + 254 - x^4 \, dx$$

$$\frac{2}{3}x^{3/2} + 254x - \frac{x^5}{5} \Big|_0^4$$

$$x\left(\frac{2}{3} \cdot x^{1/2} + 254 - \frac{x^4}{5}\right)\Big|_0^4$$

$$4\left(\frac{2}{3}\sqrt{4} + 254 - \frac{256}{5}\right) - 0 \approx 800$$

51

50

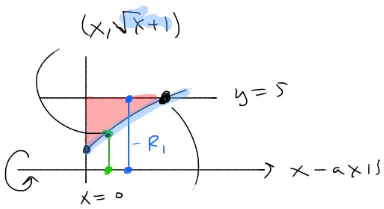
1 200

4. Consider the region bound by $x = 0$, $y = 5$ and $y = \sqrt{x} + 1$. Find the volume of the solid of revolution when the region is revolved about the:

x-axis

① b/c edge of region isn't attached to axis ⟹ washer (hole)

② slice ⊥ axis!



R_1 = touch axis & top of region = 5

R_2 = touch axis & bottom of region = $\sqrt{x} + 1$

$\sqrt{x} + 1 = 5$

$\sqrt{x} = 4$

$x = 16$

③ $A(x) = \pi(R_1^2 - R_2^2) = \pi\left(25 - (\sqrt{x} + 1)^2\right)$

④ int ALONG axis

$$\int_0^{16} \pi\left(25 - (\sqrt{x} + 1)^2\right) dx$$

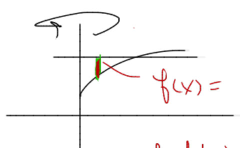
⑤ $\pi \int_0^{16} 24 - x - 2x^{1/2} \, dx$

⑥ $\pi\left(24x - \frac{x^2}{2} - \frac{4x^{3/2}}{3}\right)\Big|_0^{16}$

$16/3 = 5.3$

$= \pi x\left(24 - \frac{x}{2} - \frac{4x^{1/2}}{3}\right)\Big|_0^{16} = 16\pi\left[24 - 8 - \frac{4 \cdot 4}{3}\right] = 16\pi(10.7) \approx 160\pi$

5. Revolve the region above about the **y-axis** and compute the volume of the resulting solid.



$f(x) = 5 - (\sqrt{x} + 1) = 4 - \sqrt{x}$

shell: int ⊥ axis

$$V = \int_0^{16} 2\pi x \cdot f(x) \, dx = \int_0^{16} 2\pi x \cdot (4 - \sqrt{x})$$

$$= 2\pi \int_0^{16} 4x - x^{3/2} \, dx$$

$$= 2\pi\left(2x^2 - \frac{2}{5}x^{5/2}\right)\Big|_0^{16} \qquad 2^8$$

$$= 4\pi x^2\left(1 - \frac{1}{5}x^{1/2}\right) = 4\pi(16)^2\left(1 - \frac{4}{5}\right)$$

$$= 2^{10}\pi\left(\frac{1}{5}\right) = \frac{2^{10}\pi}{5}$$

likely on exam!

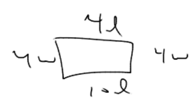
6. A box with an open top is to be constructed from a square piece of cardboard, 12 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

$V = lwh$

solve $V' = 0$

7. What are the dimensions of biggest rectangular fence (in area) you can build if you can only spend \$140 and one of the sides is to be made of stone which costs $10\frac{\$}{ft}$, and the remaining sides are to be made of wood which costs $4\frac{\$}{ft}$?

$140 = 14l + 8w$



$A = l \cdot w$

solve $A' = 0$

$\frac{140-8w}{14} = l$

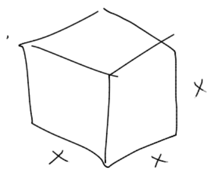
$A = l\left(\frac{140-8w}{14}\right)$

A'

chain rule

8. Suppose each edge of a cube increases at a rate of $4\frac{in}{sec}$.

(8.1) How fast is the volume growing at the instant the edge has length 5 *in*?



$$V = x^3 \qquad \frac{dx}{dt} = 4$$

$$\frac{dV}{dt} = 3\underbrace{x}_{=5}^{2} \cdot \overbrace{\frac{dx}{dt}}^{=4}$$

chain

(8.2) How fast is the volume growing at the instant the edge has length 10 *in*?