Wk 2

WW #22

$$\frac{t-t^{-1}}{t+t^{-1}} = \frac{\square}{\square}$$

$t^{-1} = \frac{1}{t}$

(★) $\dfrac{\frac{A}{B}}{\frac{C}{B}} = \dfrac{A}{C}$

$$\frac{\frac{t}{t}\cdot\frac{t}{1} - \frac{1}{t}}{\frac{t}{t}\cdot\frac{t}{1} + \frac{1}{t}} \quad \underset{\text{common denom}}{=\!=} \quad \frac{\frac{t^2-1}{t}}{\frac{t^2+1}{t}} \overset{(★)}{=} \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$$

WW #29

what's the domain

$$f(x) = \sqrt[9]{7x^2-8x}.$$

$(-\infty, \infty)$

Note the degree of the root matters

$\Rightarrow$ can't "even-root" a negative

$\Rightarrow$ you can "odd-root" a negative

Ex $\sqrt[3]{-1} = -1$

Ex: $g(x) = \sqrt[8]{7x^2-8x}$

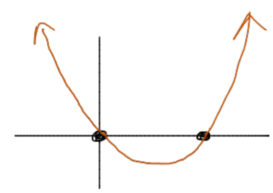
Solve: $7x^2 - 8x \geq 0$

b/c $7>0 \Rightarrow$ ∪

$x(7x-8) = 0$

$x = 0$

$x = 8/7$



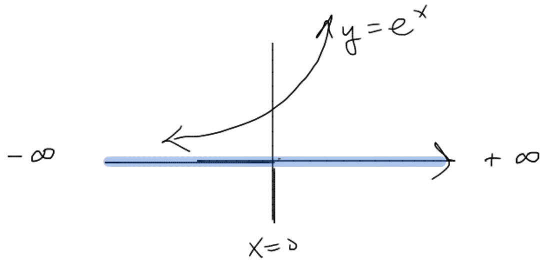
$\Rightarrow \quad (-\infty, 0] \cup [8/7, \infty)$

Limits

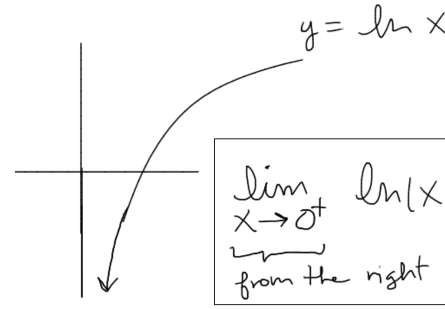
Def'n: $\lim_{x \to c} f(x) = L$ (→ some #) means you can make f(x) arbitrarily close to L, by making x close to c

$\#_x \longrightarrow L$ when $x \to c$

Fact: Limits are kinda like asymptotes



$$\lim_{x \to -\infty} e^x = 0$$



$$\lim_{x \to 0^+} \ln(x) = -\infty$$

(0^+: from the right)

Compute limits with table

$$\lim_{x \to 4} \frac{1}{(x-4)^2} = +\infty$$

x	3	3.5	3.9	3.99	3.999	4.001	4.01	4.1	4.5	5
$f(x)$	1	4	100	1000	100000	1000000	10000	100	4	1

$\frac{1}{(3-4)^2} = 1$ $\quad \frac{1}{(3.5-4)^2}$ $\quad \frac{1}{(3.9-4)^2}$ $\quad \frac{1}{(5-4)^2} = 1$

First Rule of Limits:

If, when you plug in the x, you get a real #, THAT's the limit.

Table of x's: need to approach 4.

$$\lim_{x \to 4} \frac{1}{(x-4)^3} = DNE$$

x	3	3.5	3.9	3.99	3.999	4.001	4.01	4.1	4.5	5
$f(x)$	-1	-8	-10^3	-10^6	$\approx -\infty$	$\approx +\infty$	10^6	10^3	8	1

when the left limit and right limit do no agree …
we say the limit does not exit

$$\lim_{x \to 4^-} \frac{1}{(x-4)^3} = -\infty \qquad \lim_{x \to 4^+} \frac{1}{(x-4)^3} = +\infty$$

Often limits are easy!

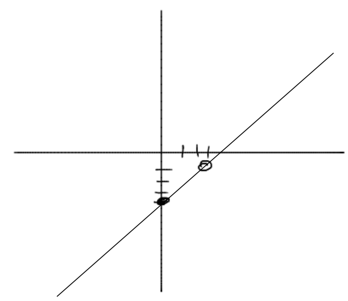
$$\lim_{x \to 2} x^2 - 7x + 12 = 2^2 - 7 \cdot 2 + 12 = 4 - 14 + 12 = 2$$

$$\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3} \overset{\text{eval}}{=} \frac{9 - 21 + 12}{0} = \frac{0}{0}$$

$x^2 - 7x + 12 = (x-3)(x-4)$

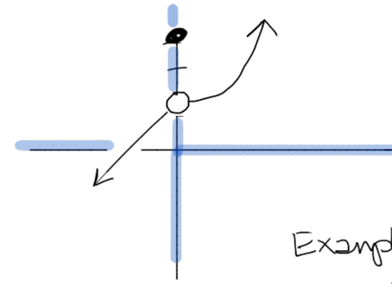
$$= \lim_{x \to 3} \frac{x-4}{1} = -1$$

$y = \frac{x^2 - 7x + 12}{x - 3}$



Piecewise Limits

$$f(x) = \begin{cases} e^x & x > 0 \\ 3 & x = 0 \\ x + 1 & \leftarrow x < 0 \end{cases}$$



Example of a discontinuous function.

Compute $\lim_{x \to 0} f(x) = \underline{1}$

① Left: $\lim_{x \to 0^-}$ $\hookrightarrow$ $(x < 0)$ $\quad \lim_{x \to 0} x+1 = 1$

② Right: $\lim_{x \to 0^+}$ $\hookrightarrow$ $x > 0$ $\quad \lim_{x \to 0} e^x = 1$

③ Agree: Y

$f(0) = 3$