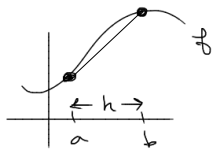
Tue Wk 2

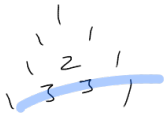
warm-up: Simplify $\dfrac{f(x+h)-f(x)}{h}$ $\leftrightsquigarrow$ $\dfrac{f(b)-f(a)}{b-a}$ (slope)



when $f(x) = 2x^3$

Recall: $f(4) = 2 \cdot 4^3$ $\qquad$ $f(\text{whatever}) = 2(\text{whatever})^3$

Function Evaluation re: Difference Quotient



$$\frac{2(x+h)^3 - 2x^3}{h} = \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$$

$$= \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3}{h} = \frac{h(6x^2 + 6xh + 2h^2)}{h}$$

$$\boxed{6x^2 + 6xh + 2h^2}$$

Inverse Functions

(1.5)

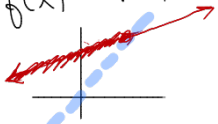
(If $f(x)$ is 1-1, it's invertible)

↑ exactly one output for each input

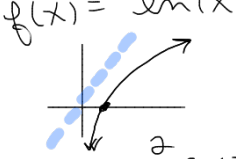
Invertible Functions

Yes

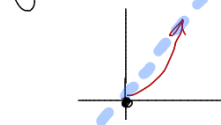
$f(x) = mx + b$ | $f^{-1}(x) = \frac{1}{m}x + C$



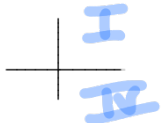
$f(x) = e^x$ | $f^{-1}(x) = \ln(x)$



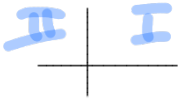
$f(x) = x^2 \{x \geq 0\}$ | $f^{-1}(x) = \sqrt{x}$



$f(x) = \sin(x) \quad \{-\frac{\pi}{2} < x < \frac{\pi}{2}\}$

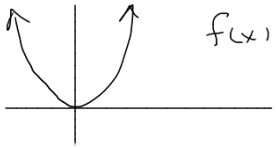


$f(x) = \cos(x) \quad \{0 < x < \pi\}$



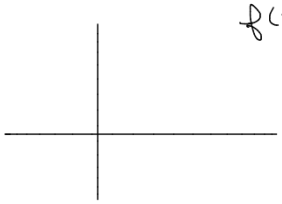
$f(x) = \boxed{\frac{ax+b}{cx+d}}$

No



$f(x) = x^2$

$f(x) = x^4 - x^2$



Exercise Finding Inverse Fcns

①	set	$y = f(x)$
②	swap	$x \leftrightarrow y$
③	solve	for y

$f(x) = 3x + 4$

$f(2) = 10$

$f(5) = 19$

$y = 3x + 4$

$x = 3y + 4$

$x - 4 = 3y$

$\frac{1}{3}x - \frac{4}{3} = y$

$f^{-1}(x) = \frac{1}{3}x - \frac{4}{3}$

$f^{-1}(10) = \frac{1}{3}(10) - \frac{4}{3} = \frac{10}{3} - \frac{4}{3} = \frac{6}{3} = 2$

$f^{-1}(19) = \frac{1}{3}(19) - \frac{4}{3} = \frac{19 - 4}{3} = 5$

$f(x) = \frac{17x - 14}{-8x + 9}$ Find $f^{-1}(x)$.

$f(1) = \frac{17 - 14}{-8 + 9} = 3$

① $y = \frac{17x - 14}{-8x + 9}$

② $x = \frac{17y - 14}{-8y + 9}$

③ (i) clear denom.

$(-8y + 9)x = 17y - 14$

(ii) distribute

$-8xy + 9x = 17y - 14$

(iii) isolate y (gather 1st)

$9x + 14 = 17y + 8xy$

$= y(17 + 8x)$

$y = \frac{9x + 14}{17 + 8x} = f^{-1}(x)$

$f^{-1}(3) = \frac{9 \cdot 3 + 14}{17 + 8 \cdot 3} = \frac{41}{41} = 1$

Tomorrow

Exp & log Fcns:

$\log_a x = y$ means $a^y = x$