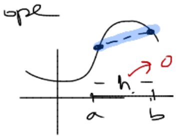
Thu. wk 2

warm-up: Simplify $\frac{f(x+h)-f(x)}{h}$ — really $\frac{f(b)-f(a)}{b-a}$ = slope



when $f(x) = 2x^3$

$f(☺) = 2(☺)^3$ $\quad = \frac{2(x+h)^3 - 2x^3}{h}$

1
1 1
1 2 1
1 3 3 1

$= \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$

$= \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3}{h} = \frac{\cancel{h}(6x^2 + 6xh + 2h^2)}{\cancel{h}} = 6x^2 + 6xh + 2h$

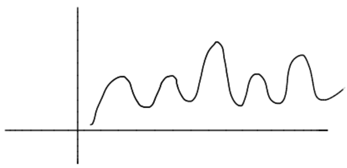
Function Evaluation re: difference quotient simplification

Today: Inverse Functions
(1.5)

Functions that are "1-1" (one-to-one) are "invertible".

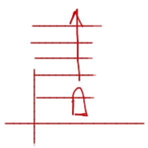
Ex. Your location @ time t is not invertible } does not satisfy horizontal line test.

$L(t) =$ location.



a given point in time doesn't correspond to a unique location.

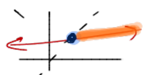
" i.e., you can visit the same location throughout the day."

Ex.  Height of rocket during take-off

Functions & Invertibility

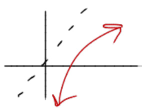
Yes

$y = mx + b$

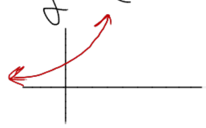


$y = \frac{1}{m}x + c$

$y = \ln(x)$



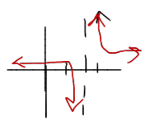
$y = e^{x}$



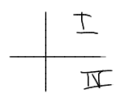
$y = x^2 \; \{x \geq 0\}$

$y = \sqrt{x}$

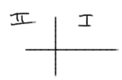
$y = \frac{x+1}{2x-3}$



$y = \sin(x) \quad \{-\frac{\pi}{2} \leq x < \frac{\pi}{2}\}$

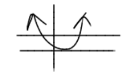


$y = \cos(x) \quad \{0 \leq x < \pi\}$

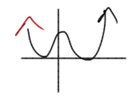


No

$y = x^2$



$y = x^4 - x^2$



Important Feature of Inverse Functions

1. (swap x & y)
2. Label $f^{-1}(x) \equiv$ f-inverse "un-does f"
3. $f^{-1}(f(x)) = x$

   $f(f^{-1}(x)) = x$

Ex:

$f(x) = 7x + 8$

Find $f^{-1}(x)$.

1. set $y = f(x)$ — $y = 7x + 8$
2. swap $x \leftrightarrow y$ — $x = 7y + 8$
3. solve for y — $x - 8 = 7y$

$y = \frac{1}{7}x - \frac{8}{7}$

$$\boxed{f^{-1}(x) = \frac{1}{7}x - \frac{8}{7}}$$

Ex.

$f(x) = \dfrac{3x + 4}{1 - x}$

$y = \dfrac{3x + 4}{1 - x}$

$x = \dfrac{3y + 4}{1 - y}$

solve

(i) clear denom $\Rightarrow$ $(1 - y) \cdot x = 3y + 4$

(ii) distribute! $x - xy = 3y + 4$

(iii) collect like terms "get all y's on same side" $x - 4 = 3y + xy$

$= y(3 + x)$

(iv) isolate y

$$\boxed{\frac{x - 4}{3 + x} = y = f^{-1}(x)}$$

Tomorrow: Exp & Log Functions

Def $\log_a x = y$ <u>means</u> $a^y = x$

logs are "just" exponents