thur. wk - 2

Logarithms & Exponentials

Def'n $\log_a X = b$ means $a^b = x$

Exercises

① $\ln(e^7) - \ln(e^6) + 5\cdot\ln(e^2) =$

$7\cdot\underbrace{\ln(e)}_{=1}$

$\underbrace{\quad}_{7} - 6 + 5\cdot 2 = 11$

② $e^{3X-1} = 5\cdot e^{1-x}$ solve for x

key: unknown variable is in exponent (upstairs)

- need it down
- hit with log $\left(\log A^c = c\cdot \log A\right)$

$\ln\left(e^{3X-1}\right) = \ln\left(5e^{1-x}\right)$

$(3X-1)\underbrace{\ln(e)}_{=1} = \ln(5) + \underbrace{\ln(e^{1-X})}_{1-x}$

$3X-1 = \ln(5) + 1 - x$

$4X = \ln(5) + 2$

$$\boxed{X = \frac{\ln(5)+2}{4}}$$

③ Solve

$$(\sqrt{2})^x = 8$$

$$\ln((\sqrt{2})^x) = \ln(8)$$

$$x \cdot \ln(\sqrt{2}) = \ln(8)$$

$$x = \frac{\ln(8)}{\ln(\sqrt{2})}$$

$$\log_2((\sqrt{2})^x) = \log_2(8) = 3$$

$$\log_2((2^{1/2})^x) = 3$$

$$\underbrace{\log_2(2^{\frac{x}{2}})} = 3$$

$$\frac{x}{2} = 3$$

$$x = 6$$

(4) $\ln(x^6) - \ln(x^3) = 3$

$$\ln\left(\frac{x^6}{x^3}\right) = 3$$

$$\ln(x^3) = 3$$

$$e^{\ln(x^3)} = e^3$$

$$x^3 = e^3$$

cube root

$$x = e$$

(1) x's are inside multiple logs

(2) combine into a single log

(3) hit w/ e

Just as

$$\ln(e^{blah}) = blah$$

$$e^{\ln(blah)} = blah$$

Application of logs

Sound:

loudness is measured in decibels ⇔ deci - bel — unit of measurement of sound named for Alex. G Bell

$\frac{1}{10}$(bel)

Decibels is a logarithmic scale

$$D = 10 \cdot \log(P/P_0)$$

(or $D = 20 \cdot \log(P/P_0)$)

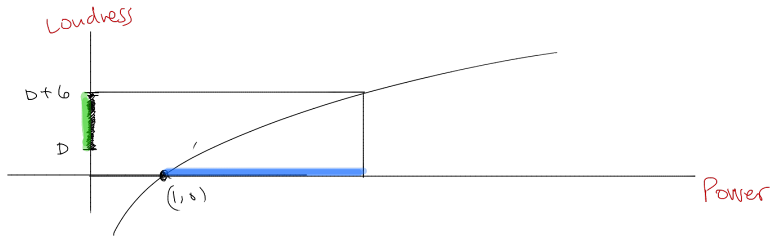
P_0 = reference power level of sound (pressure) (depends on speaker)

or = ref. amplitude of sound wave

$D = 0$ barely audible

↕ small power level

P = power level of pressure that gives D decibels



Exercise: Assume $D = 10 \cdot \log(P/P_0)$

How much more powerful is the sound to increase the decibel level by +6 dB.

(1) solve eqn above for P.

(i) $\frac{D}{10} = \log(P/P_0)$

(ii) $10^{(0.1D)} = 10^{\log(P/P_0)} = P/P_0 \Rightarrow P = 10^{(0.1D)} \cdot P_0$

How much power gives decibel level D.

(2) Examine $D + 6 = 10 \cdot \log(P_1/P_0)$ P_1 = power required to give D+6 decibels

(i) Solve for P_1

$$\frac{D+6}{10} = \log(P_1/P_0)$$

$$10^{\left(\frac{D+6}{10}\right)} = 10^{\log(P_1/P_0)} = P_1/P_0$$

$$= 10^{\left(\frac{D}{10} + \frac{6}{10}\right)}$$

$$= 10^{0.1D + 0.6}$$

$$10^{0.1D} \cdot 10^{0.6} = P_1/P_0$$

mult. by P_0

$$P_1 = 10^{0.1D} \cdot 10^{0.6} \cdot P_0$$

$$= 10^{0.6} \cdot 10^{0.1D} \cdot P_0$$

$$= 10^{0.6} \cdot P$$

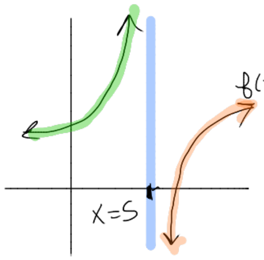
$$= 3.981 \cdot P$$

$$\approx 4P$$

So, by adding 6 decibels - we had to increase the sound by 4 TIMES as much

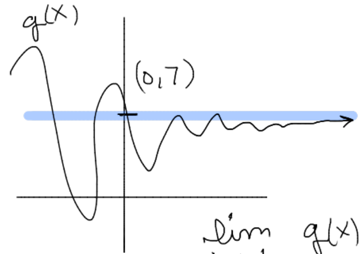
Limits

related to asymptotes



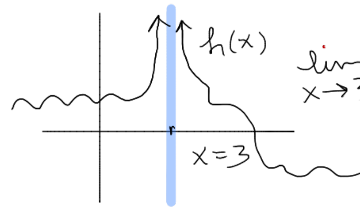
$\lim_{x \to 5} f(x)$ DNE

b/c approaching 5 from the left doesn't take us to *roughly* the same spot as approaching the right



$\lim_{x \to +\infty} g(x) = 7$ (b/c g(c) gets 'arbitrarily' close to 7 as c gets larger)

the limit as x approaches + infinity



$\lim_{x \to 3} h(x) = +\infty$

b/c as $x \to 3$ from both left and right h(x) gets infinitely larger