thur. wk 2

Logs & Exponentials

Def'n: $\log_a x = b$ means $a^b = x$

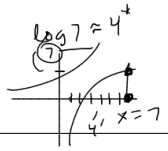
Properties

Exponents

$a^m \cdot a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^c = a^{m \cdot c}$



Note:

why $10^{\log X} = X$?

1. $\log X = \log_{10} X$ should be assumed
2. what is $\log X$ = exponent ya gotta raise 10 to, to get X
3. & $\log X$ IS an expo. fn.

Logarithms

$\log(A \cdot B) = \log A + \log B$

$\log\left(\frac{A}{B}\right) = \log A - \log B$

$c \cdot \log(A) = \log(A^c)$

(for example)

(raise both sides as power of 10)

$10^{(c \cdot \log(A))} = 10^{\log(A^c)} = A^c$

$10^{(c \cdot \log(A))} = \left(10^{\log(A)}\right)^c$

$= A^c$

Exercises:

$\ln x = \log_e x$ w/ $e \approx 2.718\ldots$

① $\ln(e^7) + \ln(e^{-1}) - 3 =$

$7 \cdot \underbrace{\ln(e)}_{=1} - 1 - 3 = 7 - 1 - 3 = 3$

② $e^{3x+7} = 15$

"variable (unknown) is in the exponent spot"

— "hit it w/ log"

$\ln(e^{3x+7}) = \ln(15)$

$3x + 7 = \ln(15)$

$x = \frac{\ln(15) - 7}{3}$

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③ $\ln(x^6) - \ln(x^3) = 3$

"unknown is inside a single log"

→ hit it w/ e

$x^{6-3} = x^3$

$\ln\left(\frac{x^6}{x^3}\right) = 3$

$\frac{x^6}{x^3} = e^{\ln\left(\frac{x^6}{x^3}\right)} = e^3 \implies x^3 = e^3 \implies (x^3)^{1/3} = (e^3)^{1/3}$

$x = e$

How does one mathematician break up with another?

$\log(\text{I'm}) - \log(\text{You}) = \log\left(\frac{\text{I'm}}{\text{You}}\right)$

(4)

$$e^{3x+5} = 5e^{1-x}$$ solve for x

"unknown is living in exponent spot"

— need to get it down

— hit w/ ln

$$\ln\left(e^{3x+5}\right) = \ln\left(5e^{1-x}\right)$$

$\ln(e^{3x+5})$: $3x+5$

mistake to bring $1-x$ down now (b/c of 5)

$$= \ln(5) + \ln\left(e^{1-x}\right)$$

$\ln(5)$: just a #; $\ln(e^{1-x})$: $1-x$

$$3x + 5 = \ln(5) + 1 - x$$

$$4x = \ln(5) - 4 \Rightarrow \boxed{x = \frac{\ln(5) - 4}{4}}$$

(5) $(\sqrt{2})^x = 8 \rightarrow (2^{1/2})^6 = 2^3$

recognize x is upstairs — needs to be down so hit w/ log

$$\ln\left(\sqrt{2}^{\,x}\right) = \ln(8)$$

$$x \cdot \ln(\sqrt{2}) = \ln(8)$$

$$x = \frac{\ln(8)}{\ln(\sqrt{2})} \stackrel{?}{=} 6$$

or

$$(\sqrt{2})^x = 8 = 2^3$$

$$\| $$

$$(2^{1/2})^x$$

$$\| $$

$$2^{\left(\frac{1}{2}x\right)} = 2^3$$

so $\frac{1}{2}x = 3 \Rightarrow x = 6$

the decibel (measure of loudness)

deci-bel b/c 1/10 of a "bel" unit of measure of sound
named for "Alexander Graham Bell"

— is on a logarithmic scale

$$D = 20 \cdot \log\left(\frac{P}{P_0}\right)$$

P_0 = constant reference level of pressure (depends on speaker)

P = level of pressure (power) required to give D decibels