Wed Wk 2

Function Composition / Evaluation / Difference Quotients

Evaluation:

whatever is between the parenthesis replaces every occurance of x

Ex

$f(x) = \sqrt{x-7}$.

Evaluate f(x) @ x = 10 means, $f(10) = \sqrt{10-7} = \sqrt{3}$

$f(y) = \sqrt{y-7}$

$f(x+5) = \sqrt{x+5-7} = \sqrt{x-2}$

$f(x^2+3) = \sqrt{x^2+3-7} = \sqrt{x^2-4}$

Composition.

$f(x) = \sqrt{x-7}$, $g(x) = \dfrac{1}{x+3}$

$f \circ g(x) \overset{\text{means}}{=\!=} f(g(x)) = \sqrt{g(x) - 7} = \sqrt{\frac{1}{x+3} - 7} = \sqrt{\frac{1-7(x+3)}{x+3}} = \sqrt{\frac{-7x-20}{x+3}}$

(domain of this is domain of result intersect with domain of g(x).)

$\mathbb{R} - \{-3\}$ and $(-7x-20)/x+3 \geq 0$

$g \circ f(x) =\!= g(f(x)) = \dfrac{1}{f(x)+3} = \dfrac{1}{\sqrt{x-7}+3}$

domain of this composition is

$\sqrt{x-7} + 3 = 0$

$\sqrt{x-7} = -3$

$x - 7 = 9$

$x = 16$

$\Rightarrow \mathbb{R} - \{16\}$

domain of result

The domain of a composite function is = the domain of the result INTERSECTED with the domain of the inside function.

Domain of Inside

$\sqrt{x-7}$, $x - 7 \geq 0$

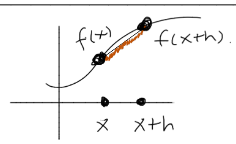
$x \geq 7$

$\boxed{[7, 16) \cap (16, \infty)}$

Domain of composite

Difference Quotient

$$\boxed{\frac{f(x+h) - f(x)}{h}}$$



(h is a small #)

slope of this segment is

Ex. Evaluate the diff. quotient when $\boxed{f(x) = \sqrt{x-7}}$

$f(x+h) = \sqrt{x+h-7}$

Ez

$f(10) = \sqrt{10-7} = \sqrt{3}$

$f(5+5) \neq \sqrt{5-7} + 5$

$f(x+h) \overset{?\ \text{NO}}{\neq} \sqrt{x-7} + h$

Difference of Squares

$A^2 - B^2 = (A-B)(A+B)$

b/c

$= A^2 + AB - BA - B^2$

$= A^2 - B^2$ ☺

① $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\sqrt{(x+h)-7} - \sqrt{x-7}}{h}$

diff. of squares

rationalize numerator ② $= \dfrac{\overset{A}{\sqrt{(x+h)-7}} \overset{-}{-} \overset{B}{\sqrt{x-7}}}{h} \cdot \left(\dfrac{\overset{A}{\sqrt{x+h-7}} \overset{+}{+} \overset{B}{\sqrt{x-7}}}{\sqrt{x+h-7} + \sqrt{x-7}}\right) = A^2 - B^2$

multiply ③ $= \dfrac{(x+h-7) \overbrace{- (x-7)}^{-x+7}}{h(\sqrt{x+h-7} + \sqrt{x-7})}$ $\underset{\text{combine}}{\overset{④}{=}}$ $\dfrac{h}{h(\sqrt{x+h-7} + \sqrt{x-7})}$ $\underset{\text{cancel}}{\overset{⑤}{=}}$ $\dfrac{1}{\sqrt{x+h-7} + \sqrt{x-7}}$

Simplify

$$\frac{x+\frac{27}{x^2}}{1+\frac{3}{x}} = \frac{\boxed{}}{\boxed{}}$$

divide by a fraction?

.. multiply by reciprocal

Idea:

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \cdot \frac{D}{C}$$

get common denom on top and again on bottom.
combine into single fractions

$$\frac{\frac{x^2}{x^2}x+\frac{27}{x^2}}{\frac{x}{x}1+\frac{3}{x}} = \frac{\frac{x^3}{x^2}+\frac{27}{x^2}}{\frac{x+3}{x}} = \frac{\frac{x^3+27}{x^2}}{\frac{x+3}{x}} = \frac{x^3+27}{x^2} \cdot \frac{x}{x+3}$$

$$= \frac{x^3+27}{x(x+3)}$$

$$= \frac{(x+3)(x^2-3x+9)}{x(x+3)}$$

$$= \frac{x^2-3x+9}{x}$$

x^3+27

$\shortparallel$

$x^3+3^3=(x+3)(x^2-3x+3^2)$

$f(x) = \frac{1}{x}$

$$f \circ f(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x$$

composition"
$\Rightarrow$ doman is domain of
intersected w/
domain of $\frac{1}{x}$

result.

$\mathbb{R} \cap (\underline{\underline{\mathbb{R} - \{0\}}})$

$$f(x) = \frac{1}{x} \quad \text{and} \quad g(x) = 8x^3 - 8x.$$

$x)$ and the domain of h.

$$f \circ g = \frac{1}{8x^3 - 8x}$$

So $8x^3 - 8x = 0$

$8x(x^2 - 1) = 0$

$x = 0$

$x = \pm 1$

$(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$