Wk 2 - Wed.

1.5.13 Computing trig / inv. trig fcns by hand

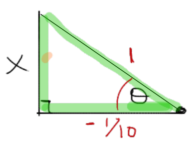
$\tan\left[\sec^{-1}(-10)\right] = ?$

① Realize $\underset{\theta}{\overset{\shortparallel}{\sec^{-1}(-10)}}$ is angle in QI or QII (b/c $\sec^{-1} = \frac{1}{\cos^{-1}}$ and $\cos^{-1}$ always gives an angle in QI or QII — by def.

② $\sec^{-1}(-10) = \theta$ compose w/ sec:

$\underbrace{\sec(\sec^{-1}(-10))} = \sec(\theta)$ $\Longrightarrow$ $\cos\theta = \frac{-1}{10}$ $\Longrightarrow$ $\boxed{\theta \text{ is in QII}}$

$-10 = \sec(\theta) = \frac{1}{\cos\theta}$

③ 

$\cos\theta = \frac{adj}{hyp}$

use Pyth. thm get other side x

$1^2 = x^2 + (-1/10)^2$

$1 - \frac{1}{100} = x^2$

$x = \pm\sqrt{1 - \frac{1}{100}} = \pm\sqrt{\frac{99}{100}} = \frac{\sqrt{99}}{\sqrt{100}}$

$= \pm\frac{3\sqrt{11}}{10}$

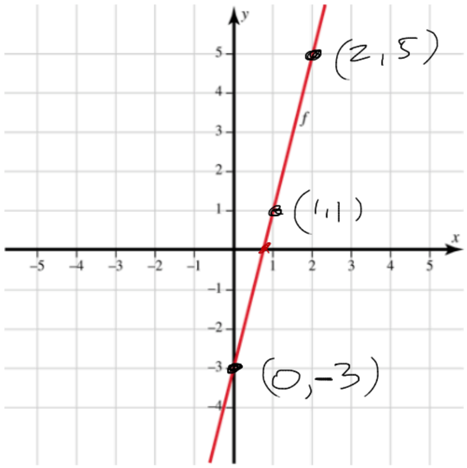
④ Since $\theta \in$ QII need $\underset{\text{opp/hyp}}{\sin\theta} > 0$ so $\frac{x}{1} > 0 \Rightarrow x > 0 = \frac{3\sqrt{11}}{10}$

⑤ $\tan(\sec^{-1}(-10)) = \tan(\theta) = \frac{opp}{adj} = \frac{\frac{3\sqrt{11}}{10}}{-\frac{1}{10}} = \boxed{-3\sqrt{11}}$

1.5

Question 2 of 13

Let f be the function in the given graph. Find f^{-1}, the inverse of f.



① Recognize $f(x)$ is linear

thus $f^{-1}(x)$ is linear

② (x, y) become (y, x)

$\Rightarrow (5,2), (1,1), (-3,0)$ be on f^{-1} graph

③ slope $\frac{2-1}{5-1} = \frac{1}{4}$

$y - 0 = \frac{1}{4}(x - (-3))$

$$\boxed{f^{-1}(x) = \frac{1}{4}x + \frac{3}{4}}$$

1.5.10 Are these inverses of each other?

$$f(t) = \frac{t+4}{t-4} \quad \text{and} \quad g(t) = \frac{4(t+1)}{t-1}$$

yes if $f \circ g(t) = t$

and $g \circ f(t) = t$

$$f \circ g(t) = f(g(t))$$

this says that EVERY occurance of t in f is replaced by g(t)

$$= \frac{\dfrac{\overbrace{4(t+1)}^{4t+4}}{t-1} + \dfrac{\overbrace{4(t-1)}^{4t-4}}{(t-1)}}{\dfrac{\overbrace{4(t+1)}^{4t+4}}{t-1} - \dfrac{\overbrace{4(t-1)}^{-4t+4}}{(t-1)}} = \frac{\left[\dfrac{8t}{(t-1)}\right]}{\left[\dfrac{8}{(t-1)}\right]} = \frac{8t}{\cancel{t-1}} \cdot \frac{\cancel{t-1}}{8} = t$$

$$g \circ f(t) = g(f(t)) = \frac{4\left(\frac{t+4}{t-4} + 1\right)}{\frac{t+4}{t-4} - 1} = \frac{\dfrac{\overbrace{4(t+4 \quad + \quad t-4)}^{4t+16+4t-16}}{t-4}}{\dfrac{t+4 \quad - \quad (t-4)}{t-4}} = \frac{8t}{8} = t$$

every occurance of t in the g-function gets replaced by f(t)

$\Rightarrow$ So yes these are inverse functions

Logs and Exponentials

$$\log_a x = b \quad \text{means} \quad a^b = x$$

Properties of Exponents — Property of Logarithm

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^c = a^{m \cdot c}$$

$$\log(A \cdot B) = \log(A) + \log(B)$$

$$\log\left(\frac{A}{B}\right) \overset{*}{=} \log(A) - \log B$$

$$\log(A^c) = c \cdot \log(A)$$

Joke: How does one mathematician break up with another?

$$\log(I'm) - \log(You) = \log\left(\frac{I'm}{You}\right)$$

Typical Use of Logs

Solve for x

$$\underbrace{\log(x-1) + \log(x+1)}_{\text{property \# 1}} = 5$$

$$\log\big((x-1)(x+1)\big) = 5$$

$$10 \qquad\qquad 10$$

blue is exponent of 10 needed to give x,
and it IS an exponent of 10, so the result is x

$$10^{\log_{10} x} = x$$

$$(x-1)(x+1) = 10^5$$

$$x^2 - 1 = 10^5$$

$$x^2 = 10^5 + 1$$

$$x = \pm\sqrt{10^5+1} \Rightarrow \text{choose } +\sqrt{10^5+1}$$

b/c of domain