Math 161 - Calculus - Exam 1 - Guide **Name:** _____

February 2, 2024

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1. Evaluate the following limits.:

(1.1) $\lim_{x \to 4} \frac{1}{x}$

(1.2) $\lim_{x \to 5} 3$

(1.3) $\lim_{x \to 4} \frac{1}{x-4}$

(1.4) $\lim_{x \to 5} \frac{-1}{(x-5)^2}$

(1.5) $\lim_{x \to 7} \frac{1}{x-4}$

(1.6) $\lim_{x \to +\infty} \frac{1}{x-4}$

(1.7) $\lim_{x \to +\infty} \frac{\cos(2x)}{x}$

(1.8) $\lim_{x \to 0} \frac{\sin(x)}{x}$

(1.9) $\lim_{x \to +\infty} e^x \cos(x)$

(1.10) $\lim_{x \to 4} \left[\frac{2}{x-4} - \frac{2}{x^2 - 7x + 12} \right]$

(1.11) $\lim_{x \to +\infty} x - \sqrt{x^2 - 6x}$

2. (Give a short written response) What does the derivative tell you about a function?

3. Use the definition of the derivative to compute $f'(x)$.

(3.1) $f(x) = \dfrac{3}{x-1}$

(3.2) $f(x) = 5\sqrt{x+2}$

4. Find all solutions

(4.1) $3e^x + 5 = e^x + 11$

(4.2) $\left(1 + \frac{0.06}{12}\right)^{2x} = 4$

(4.3) $\frac{50}{1 + 2e^{3x}} = 10$

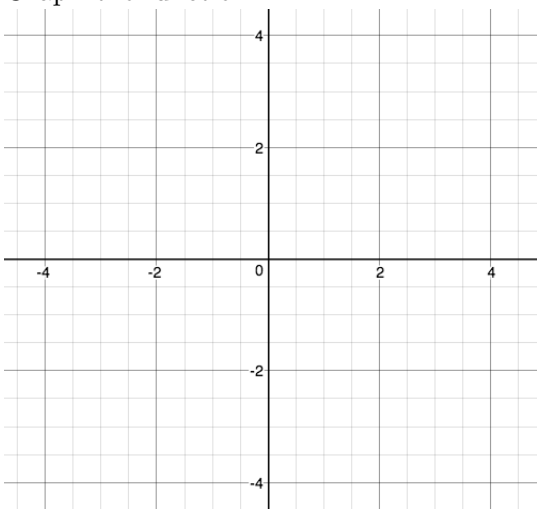
5. Find $f^{-1}(x)$.

(5.1) $f(x) = \frac{1 - 4x}{3x + 2}$

6. Given

$$f(x) = \begin{cases} 5 - x^2 & x > 2 \\ -2x + 1 & x < 2 \\ 4 & x = 2 \end{cases}$$

(6.1) Graph the function



(6.2) Finish the definition below:
A function $f(x)$ is continuous at $x = a$ if

(6.3) Use the definition of continuity to show that $f(x)$ is not continuous at $x = 2$.

(6.4) Is there a way to define $f(x)$ at $x = 2$ so that $f(x)$ is continuous at $x = 2$? Why or why not?

5. According to Peak Oil Theory, first proposed in 1956, the total amount of crude oil $Q(t)$ produced worldwide up to time t has a graph like the one shown below.

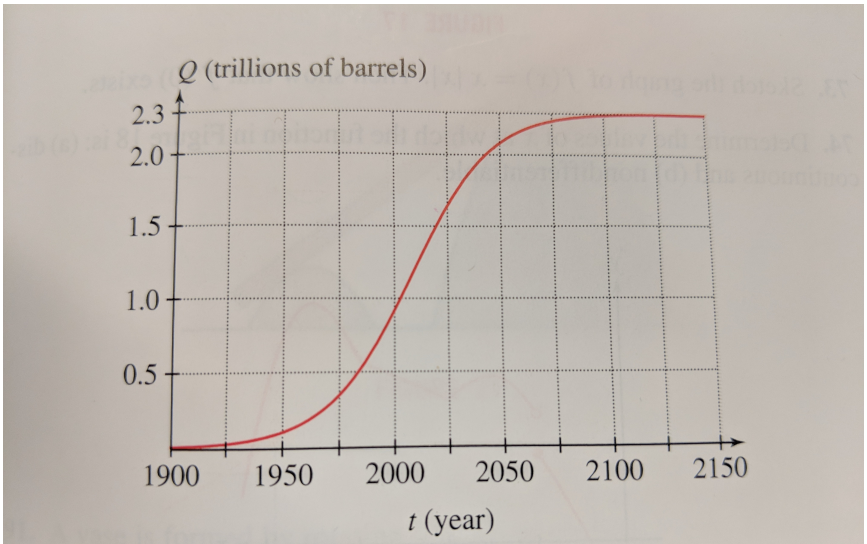


FIGURE 14 Total oil production up to time t.

(6.1) Estimate the average rate of change of oil production from 1900 to 2020.

(6.2) Estimate the instanenous rate of change of oil production at the year 2100.

(6.3) Compute and interpret $L = \lim_{t\to\infty} Q(t)$.

7. If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters after t seconds is given by $y = 58t - .83t^2$.

   (a) Find the average velocity over the given time intervals:

   (7.1) time interval: [1,1.5] _____

   (7.2) time interval: [1,1.01] _____

   (7.3) time interval: [1,1.001] _____

   (7.4) Find the instantaneous velocity after one second (to the nearest hundredth).

8. The position of a cat running from a dog down a dark alley is given by the values of the table.

t (seconds)	0	1	2	3	4	5
s (feet)	0	14	30	73	100	117

   Find the average velocity of the cat for the time period beginning with $t = 2$ and lasting

   (8.1) 3 seconds

   (8.2) 2 seconds

   (8.3) 1 seconds

   Estimate the instantaneous velocity when $t = 2$ by finding the average velocity from $t = 1$ to $t = 3$.

   Do you think this is a good estimate or not? Explain.