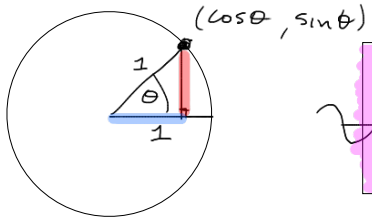
Friday - Week 3

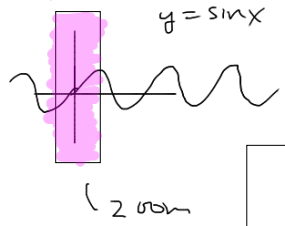
1. Study Guide up!
2. Limit exercises posted too.
3. Today: More derivative calculations and interpretations
4. WW Q's

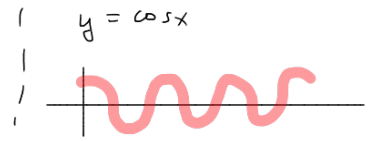
WW #7

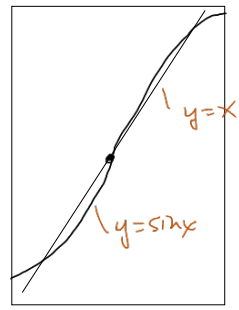
$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\approx \frac{x}{x} = 1$$









Near $x=0$

$\sin x \approx x$

Ex

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = .5$$

$= y$

x	-.1	-.01	.01	.1
y	4.9	4.99	5.1	5.01

Derivative Calculation

$$f(x) = \sqrt{x-1}$$

(diff of □'s)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h} \cdot \frac{\sqrt{x+h-1} + \sqrt{x-1}}{\sqrt{x+h-1} + \sqrt{x-1}}$$

$$= \lim_{h\to 0} \frac{x+h-1 \overbrace{-(x-1)}^{-x+1}}{h(\sqrt{x+h-1} + \sqrt{x-1}} = \lim \frac{h}{h(\sqrt{x+h-1} + \sqrt{x-1}}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h-1} + \sqrt{x-1}} = \frac{1}{\sqrt{x+0-1} + \sqrt{x-1}} = \frac{1}{\sqrt{x-1} + \sqrt{x-1}}$$

$$= \boxed{\frac{1}{2\sqrt{x-1}}}$$

property

$$\lim_{x\to c} \frac{f}{g} = \frac{\lim f}{\lim g}$$

$$\lim_{x\to c} \sqrt{f+g} = \sqrt{\lim f + \lim g}$$

Diff of □'s

① $(A-B)(A+B) = A^2 + AB - BA - B^2$

$= A^2 - B^2$

② $(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})$

$= \sqrt{x}\cdot\sqrt{x} - \sqrt{x}\sqrt{y} - \sqrt{y}\sqrt{x} - \sqrt{y}\cdot\sqrt{y}$

$= x^{\frac{1}{2}} x^{\frac{1}{2}}$ $\quad y^{\frac{1}{2}} \cdot y^{\frac{1}{2}}$

$= x - y$

Derivative of a rational function $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x-5}$, $k(x) = \frac{3}{5x-4}$

$$f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{1}{h} \cdot \left(\frac{x}{x} \frac{1}{x+h} - \frac{x+h}{x+h} \frac{1}{x} \right)$$

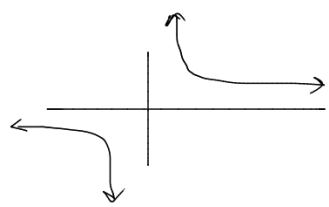
$$= \lim_{h \to 0} \frac{1}{h} \left(\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)} \right) = \lim_{h \to 0} \frac{1}{h} \left(\frac{x - (x+h)}{x(x+h)} \right) = \lim_{h \to 0} \frac{1}{\cancel{h}} \frac{-\cancel{h}}{x(x+h)}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)} = \boxed{\frac{-1}{x^2}}$$

Question:

- What is the limit of the $\overbrace{\text{slopes of the tangent line}}^{-\frac{1}{x^2}}$ of the graph as $x \to \infty$? $= 0$

- Based on this calculations describe the slope of the graph of $y = \frac{1}{x}$ as $x \to \infty$.



it flattens out