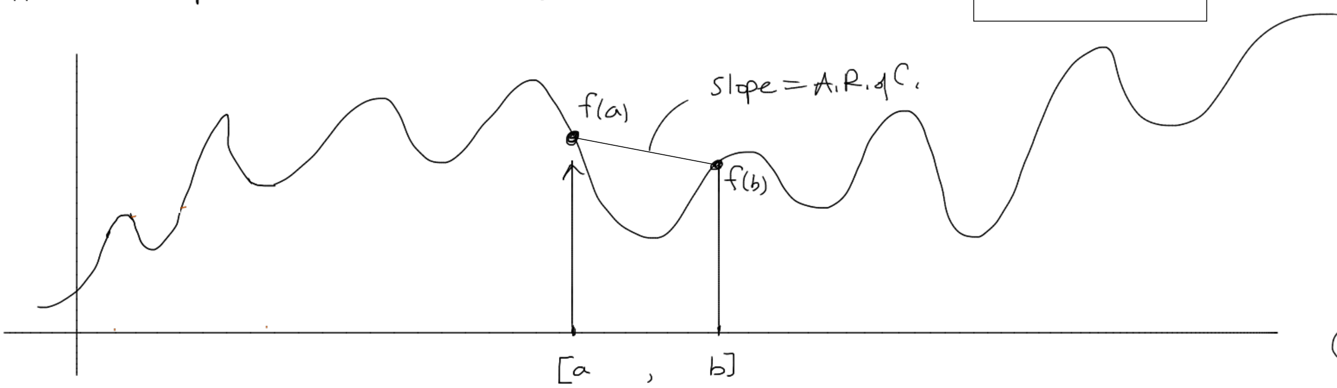
Monday - Week 3

1. Average Rate of Change
2. Continuity
3. The Derivative

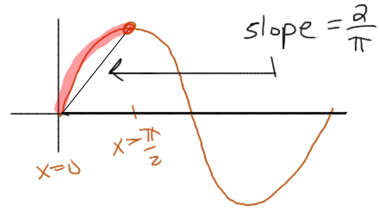
Average Rate of Change of a function on $[a,b]$, $\boxed{\dfrac{f(b)-f(a)}{b-a}}$



Ex. $f(x) = \sin x$

avg. rate of change of $f(x)$ on $[0, \pi/2]$

$$\text{A.R.ofC} = \frac{\sin(\pi/2) - \sin(0)}{\pi/2 - 0} = \frac{1-0}{\pi/2} = \frac{2}{\pi} \approx \frac{2}{3}$$



Ex $f(x) = \sin(x)$

avg rate of change on $[0, 2\pi]$

$$\frac{f(b)-f(a)}{b-a} = \frac{\sin 2\pi - \sin 0}{2\pi - 0} = \frac{0-0}{2\pi} = \emptyset$$

A few more limits

$$\lim_{x\to+\infty} \frac{\sin x}{x} = 0$$ b/c squeeze thm.

Idea: Facts: $\lim_{x\to+\infty} \frac{1}{x} = 0$

$\lim_{x\to+\infty} \frac{-1}{x} = 0$

$-1 \le \sin x \le 1$

since $x\to+\infty$, $x>0$: $\frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$

Since RHS $\lim_{x\to+\infty} \frac{1}{x} = 0$

LHS $\lim_{x\to+\infty} \frac{-1}{x} = 0$

$\Rightarrow \lim_{x\to+\infty} \frac{\sin x}{x} = 0$

squeeze theorem

$$\lim_{x\to+\infty} e^x = +\infty$$

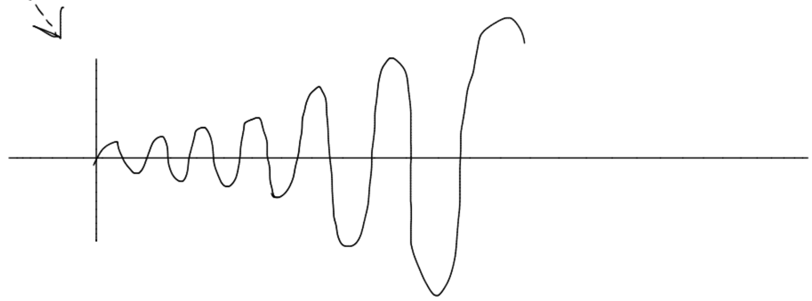
$$\lim_{x\to+\infty} \cos x = \text{DNE}$$

In order for a limit to exist, you've got approach a certain number AND STAY CLOSE

x	2π	4π	6π	6000π	10000π
$\cos x$	1	1	1	1	1

(this doesn't mean $\lim_{x\to+\infty} \cos x = 1$

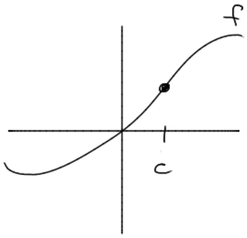
$$\lim_{x\to+\infty} e^x \cdot \cos x = \text{DNE}$$



$$\lim_{x\to 0} \frac{\sin x}{x} \approx \frac{x}{x} = 1$$

zoom in on graph of $y = \sin x$

it looks like $y = x$



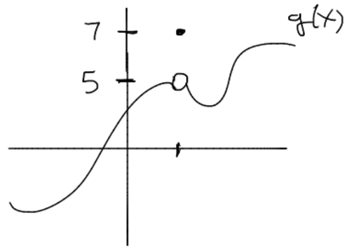
continuous @ $x=c$

Def'n

$$\lim_{x\to c} f(x) = f(c)$$

i.e.,

the limit exists and equals the value of the function @ $x=c$

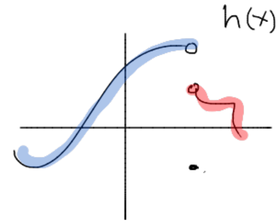


$$\lim_{x\to c} g(x) = 5$$

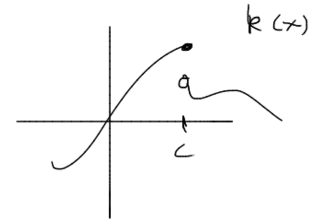
but

$$g(c) = 7$$

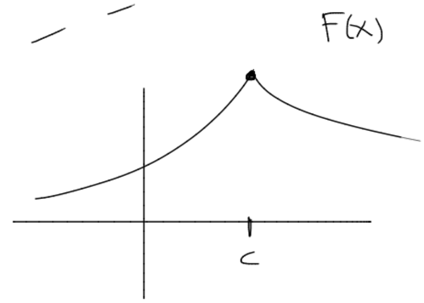
$\Rightarrow$ $\boxed{\text{not continuous}}$



$$\lim_{x\to c} h(x) = \text{DNE}$$



$$\lim_{x\to c} k(x) = \text{DNE}$$



$F(x)$ is continuous

$F(x)$ is not differentiable @ $x=c$

it's derivative @ $x=c$ DNE

Ex Is $f(x)$ continuous $x = 1$

$$f(x) = \begin{cases} e^{x-1} & x < 1 \\ x & x = 1 \\ \ln(x)+1 & x > 1 \end{cases}$$

continuous b/c they're all equal

① $\lim_{x \to 1} f(x)$

L: $\lim_{x \to 1^-} f(x) = \lim_{x \to 1} e^{x-1} \overset{\text{sub}}{=} e^{1-1} = e^0 = 1$ ($x<1$)

R: $\lim_{x \to 1^+} f(x) = \lim_{x \to 1} \ln(x)+1 = \ln(1)+1 = 0+1 = 1$ ($x>1$)

limit exists

② $f(1) = 1$