LIMITS

$\lim_{x \to c} f(x) = L$ means

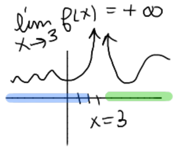
$\forall\, \epsilon > 0\ \exists\, \delta \in \mathbb{R}$ s.t.
if $|x - c| < \delta$ then
$|f(x) - L| < \epsilon$

also

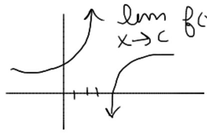
$\underbrace{\lim_{x \to c^-} f(x)}_{x \approx c \text{ but } x < c} = \underbrace{\lim_{x \to c^+} f(x)}_{x \approx c \text{ but } x > c} = L$

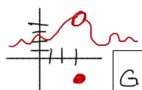
GRAPHICALLY

$\lim_{x \to 3} f(x) = +\infty$



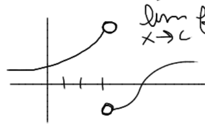
$\lim_{x \to c} f(x) = DNE$





$\lim_{x \to 3} f(x) = 4$

$\lim_{x \to c} f(x) = DNE$



Tables

$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

$\longrightarrow$ $x=0$ $\longleftarrow$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\boxed{\frac{\sin(x)}{x}}$	.998	.9999	.99999	.9999	.9999	.998

PROPERTIES

Algebra: $\lim_{x \to c}\left(f(x) + g(x)\right) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$ ($+$: $*$, $-$, $\div$, $\sqrt{\ }$)

Constants: $\lim_{x \to c} k = k$ (k is indep. of x)

$\lim_{x \to c} k \cdot f(x) = k \cdot \lim_{x \to c} f(x)$

multiplicative constants pull out in front

Ex $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} \stackrel{D.S}{=} \frac{x^2 - x^2}{0} = \frac{0}{0}$

algebra

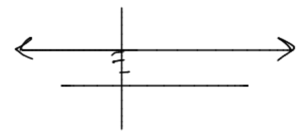
$$\lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x^2}}{h} = \lim_{h\to 0} \frac{2xh + h^2}{h} = \frac{2x \cdot 0 + 0^2}{0} = \frac{0}{0}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h\to 0} 2x+h = 2x$$

Ex Suppose $\lim\limits_{x \to 115} f(x) = 12$

Compute: $\lim\limits_{x \to 115} (f(x))^2 = \left(\lim\limits_{x \to 115} f(x)\right)^2 = 12^2 = 144$

pass limit thm



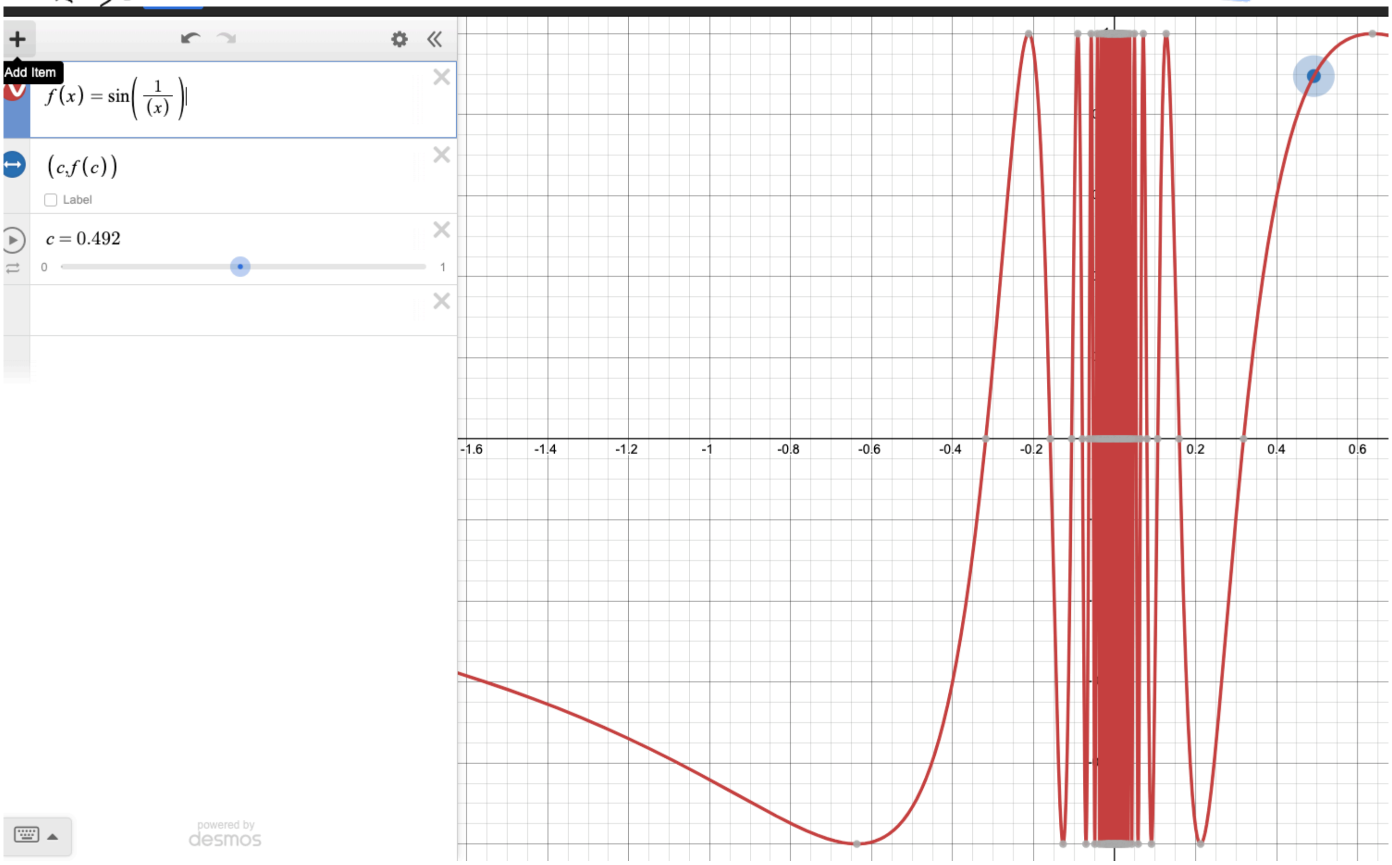
Ex Suppose $\lim\limits_{x \to 5} f(x) = 7$

1. $\lim\limits_{x \to 5} \left(f(x) + 3\right) = \underbrace{\lim\limits_{x \to 5} f(x)}_{\text{known } = 7} + \underbrace{\lim\limits_{x \to 5} 3}_{= 3}$

the height of the graph of y = 3 when x is close to 5 is, 3.

$$\lim_{x \to 0} \sin\left(\frac{1}{x}\right) = DNE$$

$+\infty$, $-\infty$, DNE, #



Compute Limits w/ Algebra

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = \lim_{x\to 3} (x+3) = 6$$

|| D.S.

$$\frac{3^2-9}{3-3} = \frac{0}{0}$$

How to compute Limits

1. Try Direct Substitution
   a. if we get a legit #, that's the limit
   b. if you get 0/0 or inf/inf, try algebraic manip
      i. try Direct Sub again

Ex

just to left of $x=2$, ht $=4$

$$\lim_{x \to 2^-} f(x) = 4$$

ht $=6$ to right of 2

$$\lim_{x \to 2^+} f(x) = 6$$

$$\lim_{x \to 4^+} f(x) = 2$$