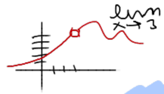
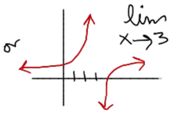


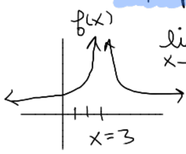
$\lim_{x\to 3} f(x) = 5$

or

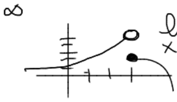


$\lim_{x\to 3} f(x) = \text{DNE}$

Graphical



$\lim_{x\to 3} f(x) = +\infty$



$\lim_{x\to 3} f(x) = \text{DNE}$

$\lim_{x\to c} k\cdot f(x) = k\cdot \lim_{x\to c} f(x)$

↑ independent of x

Properties

ALGEBRA

$\lim_{x\to c} \left(f(x) \overset{*}{\pm \div} \sqrt{\ } g(x)\right) = \lim_{x\to c} f(x) \overset{*}{\pm \div} \sqrt{\ } \lim_{x\to c} g(x)$

$\lim_{x\to c} k = k$ (b/c no x in k)

$\boxed{\text{LIMITS}}$

$\lim_{x\to c} f(x) = L$ means by pushing x close enough to c, you can make f(x) arbitrarily close to L ← a number

def: $\forall$ $\epsilon > 0$ $\exists$ $\delta > 0$ s.t. if $|x-c| < \delta$ then $|f(x) - L| < \epsilon$

$\forall$: for all; ϵ, δ: real #'s

→ this means:

left limit: $\underbrace{\lim_{x\to c^-} f(x)}$ = right limit: $\underbrace{\lim_{x\to c^+} f(x)}$ = L

$x\to c^+$ → $x \approx c,\ x > 0$

$x\to c^-$: "as $x \approx c$ but $x < 0$

left | right

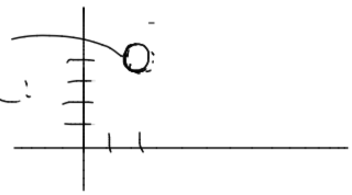
chart

x	$-\frac{1}{2}$	$-\frac{1}{10}$	$-\frac{1}{100}$	-0.00001
f(x)	-2	-10	-100	-10000

Consider the one-sided limits.

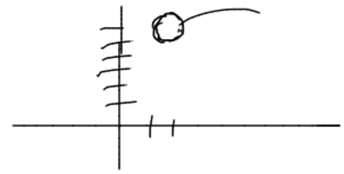
$$\lim_{x \to 2^-} f(x) = 4$$

see:

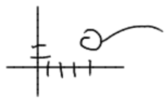


$$\lim_{x \to 2^+} f(x) = 6$$

see:



$$\lim_{x \to 4^+} f(x) = 2$$



Select a graph of f that satisfies the one-sided limits.

Ex Assume

$$\lim_{x \to 5} f(x) = 7$$

compute ①

$$\lim_{x \to 5} \left(f(x) + 3 \right) = \lim_{x \to 5} f(x) + \lim_{x \to 5} 3 = 7 + 3 = 10$$

②

$$\lim_{x \to 5} \left(3 \cdot f(x) \right) = 3 \lim_{x \to 5} f(x) = 3 \cdot 7 = 21$$

③

$$\lim_{x \to 5} \frac{x}{f(x)} = \frac{\lim_{x \to 5} x}{\lim_{x \to 5} f(x)} = \frac{5}{7}$$

Basic Limits to remember

$$\lim_{x \to +\infty} \frac{1}{x} = 0$$

#, $+\infty$, $-\infty$, DNE

chart

x	$-\frac{1}{2}$	$-\frac{1}{10}$	$-\frac{1}{100}$	-0.00001
$f(x)$	-2	-10	-100	-10000

$\to -\infty$

$\frac{1}{-1/2}$ $\quad \frac{1}{(-1/10)}$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \to 0^+} \frac{1}{x} = +\infty$$

$$\lim_{x \to 0} \frac{1}{x} = \text{DNE} \quad \text{b/c Left lim} \neq \text{Right lim}$$

Algebraic computation of limits

How to compute limits

1. Try direct sub
   a. if you get a legit #, you're done, that's the limit
   b. if not, try algebraic manipulation to simplify
   c. try direct sub again

EX

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{0}{0}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \lim_{x\to 3} (x+3) = 6$$

Tomorrow

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \overset{\text{algebra}}{=} \qquad = \text{take limit}$$