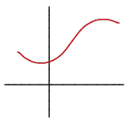
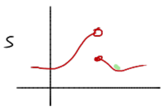
Tue - Wk 3

Continuity:

 vs 

Def: A function $f(x)$ is continuous @ $x=c$ if

$$\lim_{x \to c} f(x) = f(c)$$

(1) $\lim_{x \to c} f(x)$ exists (a #)

(2) $f(c)$ exist (a #)

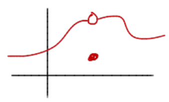
(3) these #'s are equ

Recall: $\lim_{x \to c} f(x) = L$ means left limit = right limit

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

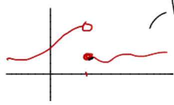
If $f(x)$ is not continuous we call it — discontinuous.

Kinds of discontinuity

(1) removable: 

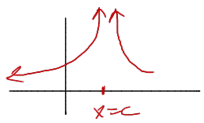
if $f(x)$ can be re-defined @ a single x-value & then be continuous,

$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

(2) jump (essential) 

[right-continuous]

$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$ yet both are real #'s

(3) infinite 

$x=c$

$\lim_{x \to c} f(x) = \infty$

Ex

$f(x) = 8(x+4)^{-6/7} - 7x^2$

Q: where is f(x) cts?

$= \dfrac{8}{(x+4)^{6/7}} - 7x^2$

↳ 7th root, to 6th power

Examine where denom = 0

I. $(x+4)^{6/7} = 0$

$\left[(x+4)^{6/7}\right]^{7/6} = [0]^{7/6}$

$x+4 = 0 \Rightarrow \boxed{x=-4}$

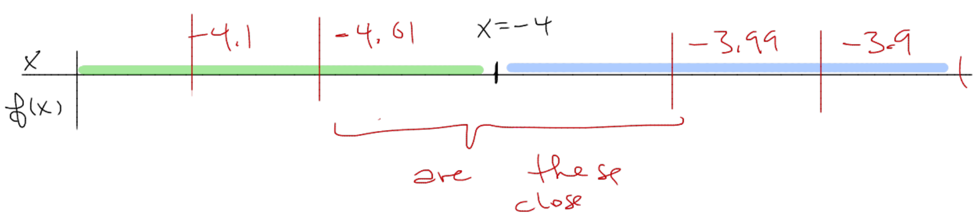
discontinuous here

Common Functions

1. polynomials: always continuous everywhere
2. rational functions: $\dfrac{f(x)}{g(x)}$ cts except for when $g(x)=0$
3. $\sin(x)$, $\cos(x)$ cts
4. $\sin\left(\frac{1}{x}\right)$ discontinuity @ $x=0$
5. $\log(x)$ & e^x are cts

II. Kind?

$\lim\limits_{x\to -4} \dfrac{8}{(x+4)^{6/7}} - 7x^2$



$\lim\limits_{x\to -4^-} \dfrac{8}{(x+4)^{6/7}} - 7x^2 \approx \dfrac{8}{\oplus(-4.01+4)^{6/7}} - 7(-4)^2$

sub $x=-4.01$

vs

$\lim\limits_{x\to -4^+} \dfrac{8}{(x+4)^{6/7}} - 7x^2 \approx \dfrac{8}{\oplus(-3.99+4)^{6/7}} - 7(-3.9)^2$

sub $x=-3.99$

b/c even 6th power this limits agree

$\lim\limits_{x\to -4^-} f(x) = \lim\limits_{x\to -4^+} f(x) = +\infty$

→ infinite discont.

$\lim_{x\to 1} f(x) = 6$ but $f(1) = -1$ $\Rightarrow$ removable discontinuous

$\neq$

$$\lim_{x\to 1^-} f(x) = 6 \qquad \lim_{x\to 1^+} f(x) = 6$$

$$\lim_{x\to 2^-} f(x) = 2 \qquad \lim_{x\to 2^+} f(x) = 2$$

$$f(1) = -1 \qquad f(2) = 2$$

$\rightarrow$ $f(x)$ is cts @ $x = 2$

Ex

$$f(x) = \frac{x-5}{|x-5|}$$

where is $f(x)$ discontinuous?

$|x-5| = 0$

means

$x - 5 = 0$

or

$-(x-5) = 0$

$\Big\}$ $x = 5$

discontinuity here

$$\lim_{x \to 5^-} f(x) \approx \frac{4.9-5}{|4.9-5|} = \frac{-0.1}{|-0.1|} = -1$$

left limit

$\|$ jump (essential)

$$\lim_{x \to 5^+} f(x) \approx \frac{5.1-5}{|5.1-5|} = \frac{0.1}{|0.1|} = 1$$

Determine constants A and B such that the given piecewise function is continuous for all x.

$$f(x) = \begin{cases} 1 & \text{if } x < -1 \\ Ax + 5 \ (4x+5) & \text{if } -1 \le x \le 2 \ \checkmark \\ -x^2 + Bx + 11 & \text{if } x > 2 \end{cases}$$

Focus on boundary #'s $x = -1$, $x = 2$

Cts needs @ $x=-1$

$$\underbrace{\lim_{x \to -1^-} f(x)}_{1} = \underbrace{\lim_{x \to -1^+} f(x) = f(-1) = A(-1) + 5}_{A(-1)+5}$$

so $A = 4$

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cts @ $x = 2$ needs

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) = 4(2) + 5 = 13$$

— applying def'n of cts —

$$\lim_{x \to 2^+} -x^2 + Bx + 11 = -4 + 2B + 11 = 2B + 7$$

sub $x = 2$

$13 = 2B + 7$

$6 = 2B$

$3 = B$