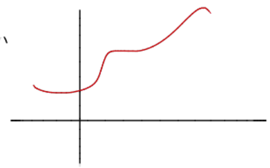
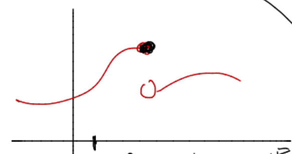
Tue wk 3

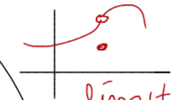
Continuity:



vs



limit DNE



limit exists but $\neq f(c)$

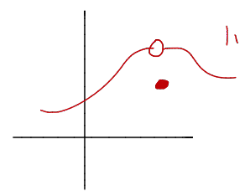
Def'n: A function $f(x)$ is continuous (cts) at $x=c$ if $\boxed{\lim_{x\to c} f(x) = f(c)}$

3 hidden assumptions
(1) limit exists (a real #) @ $x=c$
(2) $f(c)$ exists (a real #)
(3) these two #'s are equal

If $f(x)$ is not continuous @ $x=c$ then it's discontinuous

Types:

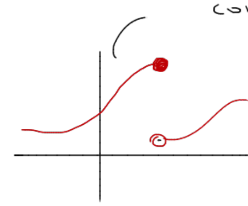
- removable discontinuity
  by redefining fcn @ a single x-value — it becomes cts



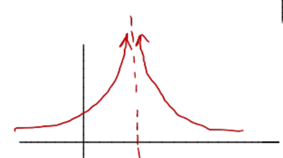
limit exists

ex: left-continuous fcn

- jump discontinuity (essential)

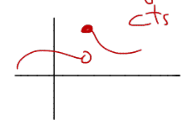


- infinite discontinuity
  $\lim_{x\to c} f(x) = \pm\infty$



$x=c$

Right cts



Reminder

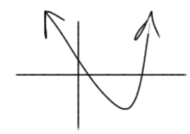
left limit = right limit

$\lim_{x\to c} f(x) = L$ means $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$

Def'n A function is continuous on (a,b) if it is cts for all $x \in (a,b)$

Ex: Polynomials are cts everywhere.

$f(x) = 3x^2 + x + 1$



Ex Rational fcns, eg., $\frac{f(x)}{g(x)}$ are continuous except where $g(x) = 0$.

Ex $\sin(x)$, $\cos(x)$ are cts everywhere

Ex $\sin\left(\frac{1}{x}\right)$ is cts away from $x = 0$

Ex e^x, $\ln(x)$ are cts.

Ex

$f(x) = 4(x+7)^{-5/3} - 2x^3 = \dfrac{4}{(x+7)^{5/3}} - 2x^3$

(denom $= 0$, so $(x+7)^{5/3} = 0 \Rightarrow x+7=0$ or $x=-7$ makes denom $=0$

$(x+7)^{5/3} = \left[(x+7)^{1/3}\right]^5$ (odd) cube roots of neg's are ok

$\Rightarrow$ cts on $(-\infty, 0) \cup (0, \infty)$, discontinuity @ $x = -7$

To classify the discontinuity:

$\lim\limits_{x \to -7} \dfrac{4}{(x+7)^{5/3}} - 2x^3 =$ ______

$x=-7$

x	-7.1	-7.01	-6.99	-6.9
f(x)		$-\infty$	$+\infty$	

infinite discontinuity

$-$ $\dfrac{4}{(-7.1+7)^{5/3}}$ vs $+$ $\dfrac{4}{(-6.9+7)^{5/3}}$ b/c of odd power these will be different

Ex Determine where $f(x)$ is cts.

$\boxed{\text{discontinuous @ } x = -7}$

(jump)

$$f(x) = \frac{x+7}{|x+7|}.$$

(1) look where denom $= 0$, $x = -7$

— everywhere else is OK.

$$\lim_{x \to -7} \frac{x+7}{|x+7|} = ???! \quad \text{look @ L \& R limits}$$

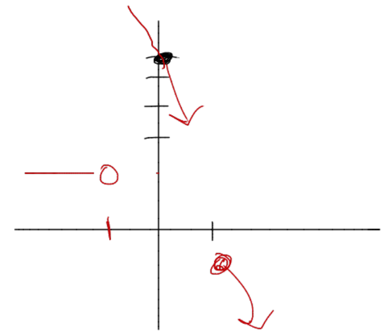
$$\lim_{x \to -7^-} \frac{x+7}{|x+7|} \approx \frac{-7.01+7}{|-7.01+7|} = \frac{-0.1}{0.1} = -1$$

$$\lim_{x \to -7^+} \frac{x+7}{|x+7|} \approx \frac{-6.99+7}{|-6.99+7|} = +1$$

-7.01 -7 -6.99

<u>Ex</u> Determine constants s.t. $f(x)$ is cts ______

$$f(x) = \begin{cases} 1 & \text{if } x < -1 \\ Ax+5 & \text{if } -1 \le x \le 2 \\ -x^2+Bx+11 & \text{if } x > 2 \end{cases}$$



<u>Focus</u>: junction pts $x=-1$ & $x=2$

Need $\lim_{x \to -1} f(x) = f(-1)$

$f(-1) = Ax+5$ w/ $x=-1$

top $(x<-1)$: $\lim_{x \to -1^-} f(x) = 1$

mid: $\lim_{x \to -1^+} f(x) = Ax+5$

$\Rightarrow Ax+5 = 1$

$-A+5 = 1$

$A=4$

Repeat for $x=2$

$\lim_{x \to 2} f(x) = f(2) = A(2)+5 = 4 \cdot 2 + 5 = 13$

$\lim_{x \to 2} f(x) = -x^2 + Bx + 11$ w/ $x=2$