Wednesday - Week 3

1. Exam 1 is NEXT Thursday - Feb. 8
2. Today - Limits & Derivatives

warm-up:

compute ① $\boxed{\lim_{x\to\infty} \frac{1}{x-4} = 0}$

use table or: set $w = x-4$ if $x\to\infty$ then $w\to\infty$

$$\lim_{x\to\infty} \frac{1}{x-4} = \lim_{w\to\infty} \frac{1}{w} = 0$$

② $\boxed{\lim_{x\to 4} \frac{1}{x-4} \quad \text{DNE}}$

Direct Sub $\frac{1}{0} \Rightarrow ?$

Check L + R limits $\left\{ \begin{array}{l} \lim_{x\to 4^-} \frac{1}{x-4} = -\infty \\ \lim_{x\to 4^+} \frac{1}{x-4} = +\infty \end{array} \right.$

Left

x	3.9	3.99	3.999
$\frac{1}{x-4}$	$\frac{1}{-.1}$ ss -10	$\frac{1}{-.01}$ ss -100	$\frac{1}{-.001}$ ss -1000

Right

x	4.1	4.01	4.001
$\frac{1}{x-4}$	$\frac{1}{.1}$ ss 10	$\frac{1}{.01}$ ss 100	$\frac{1}{.001}$ ss 1000

Reminder: A limit exists ONLY when the left limit equals the right limit

③ $\lim_{x\to 4} \frac{1}{(x-4)^2} \overset{D.S}{=} \frac{1}{0} \Rightarrow ?$ $\boxed{+\infty}$

L

x	3.9	3.99	3.999
$\frac{1}{(x-4)^2}$	$\frac{1}{(-.1)^2}$ ss 100	$\frac{1}{(-.01)^2}$ ss 10000	1000000

$\longrightarrow +\infty = \lim_{x\to 4^-} \frac{1}{(x-4)^2}$

agree

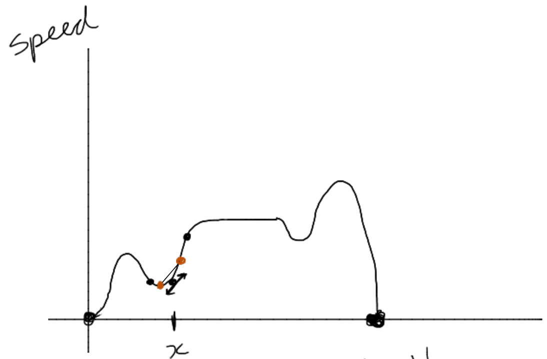
R

x	4.1	4.01	4.001
$\frac{1}{(x-4)^2}$	$\frac{1}{(.1)^2}$ ss 100	10000	1000000

$\longrightarrow +\infty = \lim_{x\to 4^+} \frac{1}{(x-4)^2}$

Derivatives: "Limit of the average rate of change" on an interval as the interval gets smaller.

Idea



How fast are we going @ point x?

First, Avg. Rate of Change of $f(x)$ on $[a,b]$

$$\frac{f(b)-f(a)}{b-a}$$

$$\boxed{\lim_{b \to a} \frac{f(b)-f(a)}{b-a}} = \text{derivative of } f(x) \text{ @ } x=b$$

replace b w/ $x+h$ (h = length of interval)

a w/ x

$$\lim_{x+h \to x} \frac{f(x+h)-f(x)}{x+h-x} = \boxed{\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}}$$

$$= f'(x)$$

Ex

$$f(x) = x^2 + 1$$

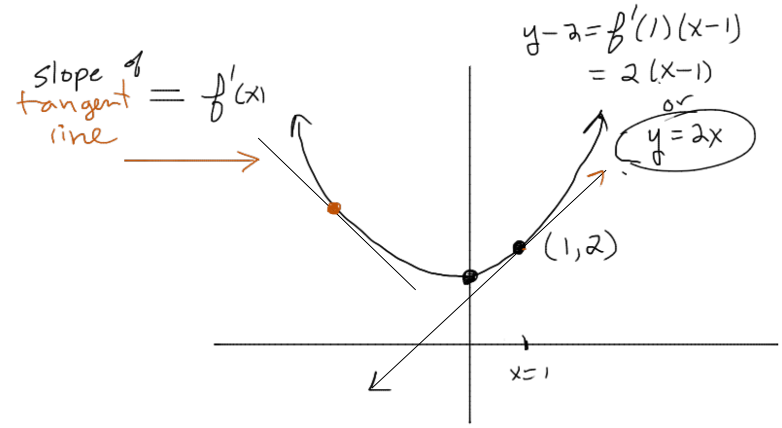
Compute

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \to 0} 2x + h = 2x$$



Ex. If $f(x) = x^2 + 1$

we found $f'(x) = 2x$

this means the slope of the tangent line to curve @

Ex.

1

1 1

1 2 1

1 3 3 1

degree sum is constant

$$f(x) = x^3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 = 3x^2$$