DIFFERENCE QUOTIENT LIMITS

(1) $\lim\limits_{h\to 0} \dfrac{5(x+h)^2 - 5x^2}{h} \overset{\text{dis.}}{=} \dfrac{0}{0}$

// crux (expand)

$5\cdot\lim\limits_{h\to 0} \dfrac{x^2+2xh+h^2-x^2}{h} = 5\lim\limits_{h\to 0} \dfrac{2xh+h^2}{h} = 5\lim\limits_{h\to 0} \dfrac{\cancel{h}[2x+h]}{\cancel{h}} = 5\lim\limits_{h\to 0}(2x+h)$

$\overset{\text{dis}}{=} 5\cdot(2x+0)$

$= 10x$

---

(2) $\lim\limits_{h\to 0} \dfrac{3\sqrt{x+h} - 3\sqrt{x}}{h} = 3\cdot\lim\limits_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h} \underset{\text{crux}}{=}$ (conjugate radical)

$= 3\cdot\lim\limits_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = 3\cdot\lim \dfrac{(x+h)\overbrace{-\sqrt{x+h}\sqrt{x}+\sqrt{x}\sqrt{x+h}}^{=0}-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= 3\cdot\lim\limits_{h\to 0} \dfrac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = 3\cdot\lim\limits_{h\to 0} \dfrac{1}{(\sqrt{x+h}+\sqrt{x})} \overset{\text{dis.}}{=} 3\cdot\dfrac{1}{\underbrace{\sqrt{x+0}}_{\sqrt{x}}+\sqrt{x}} = \boxed{\dfrac{3}{2\sqrt{x}}}$

---

crux common den

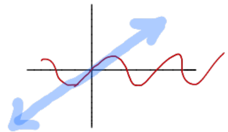
(3) $\lim\limits_{h\to 0} \dfrac{\left(\frac{5}{x+h} - \frac{5}{x}\right)}{h} \dfrac{\left(\frac{1}{h}\right)}{\left(\frac{1}{h}\right)} = \lim\limits_{h\to 0}\left[\dfrac{5}{x+h} - \dfrac{5}{x}\right]\cdot\dfrac{1}{h}$

$= \lim\limits_{h\to 0}\left[\dfrac{5}{x+h}\cdot\dfrac{x}{x} - \dfrac{5}{x}\cdot\left(\dfrac{x+h}{x+h}\right)\right]\cdot\dfrac{1}{h}$

$= \lim\limits_{h\to 0}\left[\dfrac{\overbrace{5x-5x}^{=0}-5h}{x(x+h)}\right]\cdot\dfrac{1}{h} = \lim\limits_{h\to 0}\dfrac{-5\cancel{h}}{x(x+h)}\;\dfrac{1}{\cancel{h}} = \lim\limits_{h\to 0}\dfrac{-5}{x(x+h)} \overset{\text{dis.}}{=} \dfrac{-5}{x^2}$

Important LIMITS to Remember

$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1$$



when $x \approx 0$, $\sin(x) \approx x$

1st cousin

key: { limit props: $\lim_{x\to 0} k\cdot g(x) = k\cdot \lim_{x\to 0} g(x)$

① $\lim_{x\to 0}\frac{\sin(x)}{5x} = \frac{1}{5}\cdot\frac{\sin x}{x}$

$$\frac{1}{5}\underbrace{\lim_{x\to 0}\frac{\sin(x)}{x}}_{=1} = \frac{1}{5}\cdot 1 = \frac{1}{5}$$

Key: Really think $\lim_{w\to 0}\frac{\sin(w)}{w}$ & if $x\to 0$ then $5x\to 0$

② $\lim_{x\to 0}\frac{\sin(5x)}{x} =$ so ... set $w = 5x$

$$= \lim_{x\to 0}\frac{5}{1}\cdot\frac{\sin(5x)}{5x}$$

some variable

$$= 5\cdot\underbrace{\lim_{w\to 0}\frac{\sin(w)}{w}}_{=1} = 5$$

③ $\lim_{x\to 0}\frac{\sin(13x)}{21x} = \frac{1}{21}\cdot\lim_{x\to 0}\frac{\sin(13x)}{x} = \frac{13}{21}$

$$= \frac{1}{21}\cdot 13\lim_{x\to 0}\frac{1}{13}\cdot\frac{\sin(13x)}{x} = \frac{13}{21}\underbrace{\lim_{x\to 0}\frac{\sin(13x)}{13x}}_{=1}$$

Impt. Limit to Remember || Don't forget to Direct Sub

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0$$

Numerator approaches 0
much faster than denom

① $$\lim_{x \to 0} \frac{5 - 5\cos(x)}{3x} = \lim_{x \to 0} \frac{5(1 - \cos x)}{3 \quad x} = \frac{5}{3} \underbrace{\lim_{x \to 0} \frac{1 - \cos x}{x}} = 0$$

② $$\lim_{x \to \frac{\pi}{2}} \frac{1 - \cos(x)}{7x} = \frac{1}{7} \cdot \lim_{x \to \pi/2} \frac{1 - \cos x}{x} = \frac{1}{7}\left(\frac{1 - \overbrace{\cos \pi/2}^{=0}}{\pi/2}\right) = \frac{2}{7\pi}$$