2.6

MA 161 - Wk3 - W

1. difference quotient limit
2. algebra required limit
3. rationalize required limit
4. common denom limit
5. ac factoring limit
6. piecewise limit
7. sin(27pi/4) = sin(?)

(Algebra required limits)

Ex $\lim_{h\to 0} \frac{2(x+h)^2 - 2x^2}{h} \overset{\text{factor?}}{=} \lim_{h\to 0} \frac{2[(x+h)^2 - x^2]}{h} = 2\cdot \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$

crux
(expand)

$= 2 \cdot \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = 2 \cdot \lim_{h\to 0} \frac{h(2x+h)}{h} = 2 \cdot \lim_{h\to 0} 2x+h = 2(2x+0) = 4x$

direct sub

Ex $\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \overset{\text{d.s.}}{=} \frac{0}{0}$ (keep going w/ algebra)

crux // rationalize numerator

$(A-B)(A+B) = A^2 - B^2$

$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h\to 0} \frac{\overset{F}{(x+h)} + \underbrace{\overset{O}{\sqrt{x}\sqrt{x+h}} - \overset{I}{\sqrt{x}\sqrt{x+h}}} - \overset{L}{x}}{h(\sqrt{x+h} + \sqrt{x})}$

$= \lim \frac{(\cancel{x}+h) - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$

// d.s.

$\frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}}$

Ex $\lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} \overset{\text{d.s.}}{=} \frac{0}{0}$

crux //

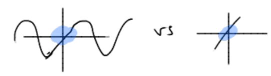
$\lim_{h\to 0} \frac{\frac{3}{x+h}\cdot\frac{x}{x} - \frac{3}{x}\cdot\frac{x+h}{x+h}}{h} = \lim \frac{\frac{3x}{(x+h)x} - \frac{3(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{\left[\frac{3x - 3(x+h)}{(x+h)x}\right]}{h}$

common denom

$= \lim_{h\to 0} \left[\frac{3x - 3x - 3h}{x(x+h)}\right] \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-3\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-3}{x(x+h)} \overset{\text{d.s.}}{=} \frac{-3}{x^2}$

Two important limits to remember (& their "cousins")

1. $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} = 1$ b/c near $x \approx 0$ $\sin(x)$ acts like x



use limit properties $\left\{ \lim\limits_{x \to 0} k \cdot f(x) = k \cdot \lim\limits_{x \to 0} f(x) \right\}$

1st cousin

$$\lim_{x \to 0} \frac{\sin(x)}{5x} = \lim_{x \to 0} \frac{1}{5} \cdot \frac{\sin(x)}{x} = \frac{1}{5} \lim_{x \to 0} \frac{\sin x}{x} = \frac{1}{5} \cdot 1 = \frac{1}{5}$$

idea: In 1 equiv, $\lim\limits_{w \to 0} \dfrac{\sin(w)}{w} = 1$

2nd cousin

$$\lim_{x \to 0} \frac{\sin(3x)}{x} = \lim_{x \to 0} \cdot \frac{3}{1} \frac{\sin(3x)}{3x} = \frac{3}{1} \cdot \lim_{x \to 0} \frac{\sin 3x}{3x}$$

since $x \to 0$ forces $3x \to 0$

set $w = 3x$

$$\frac{3}{1} \cdot \underbrace{\lim_{w \to 0} \frac{\sin(w)}{w}}_{1} = \frac{3}{1} \cdot 1 = 3$$

Ex. $$\lim_{x \to 0} \frac{\sin(15x)}{7x} = \frac{1}{7} \lim_{x \to 0} \frac{\sin(15x)}{x} = \frac{15}{7} \lim_{x \to 0} \frac{\sin(15x)}{15x}$$

so $x \to 0$, $15x \to 0$

$$= \frac{15}{7} \underbrace{\lim_{15x \to 0} \frac{\sin(15x)}{15x}}_{=1} = \frac{15}{7}$$

Ex

$$\lim_{x\to 0} \frac{\sin(x)\cdot\sec(x)}{6x} \overset{d.s.}{=} \frac{\sin(0)\cdot\sec(0)}{6\cdot 0} = \frac{0\cdot 1}{0} = \frac{0}{0}$$

Hint: use your tools ($\lim_{x\to 0} \frac{\sin(kx)}{kx} = 1$ & $\lim_{x\to 0}[f(x)\cdot g(x)] = \lim f(x)\cdot \lim g(x)$

$$= \frac{1}{6}\lim_{x\to 0} \frac{\sin(x)}{x}\cdot\frac{\sec(x)}{1} = \frac{1}{6}\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sec(x)}{1} = \frac{1}{6}\underbrace{\lim_{x\to 0}\frac{\sin x}{x}}_{=1}\cdot\underbrace{\lim_{x\to 0}\frac{1}{\cos x}}_{d.s.}$$

$$1 = \frac{1}{1} = \frac{1}{\cos(0)}$$

$$= \frac{1}{6}$$

Don't Forget Direct Sub!

Important Limit to Remember

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x} = \frac{\text{very small}}{\text{merely small}} \approx \frac{0}{1} = 0$$

Ex

$$\lim_{x \to 0} \frac{1 - \cos(x)}{5x} = \frac{1}{5} \lim_{x \to 0} \frac{1 - \cos x}{x} = \frac{1}{5} \cdot 0 = 0$$

x-coord @ 90°

Ex.

$$\lim_{x \to \pi/2} \frac{1 - \cos(x)}{7x} = \frac{1}{7} \lim_{x \to \pi/2} \frac{1 - \cos x}{x} = \frac{1}{7} \cdot \left[\frac{1 - \cos \frac{\pi}{2}}{\frac{\pi}{2}} \right] = \frac{1}{7} \left[\frac{1}{\pi/2} \right] = \frac{2}{7\pi}$$