Math 161 - Calculus - Exam 1 - Guide **Name:** _____

September 12, 2024

On the exam you must show your work to receive full credit.

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1. Evaluate the following limits.:

(1.1) $\lim_{x\to 4} \frac{1}{x}$

(1.2) $\lim_{x\to 5} 3$

(1.3) $\lim_{x\to 4} \frac{1}{x-4}$

(1.4) $\lim_{x\to 5} \frac{-1}{(x-5)^2}$

(1.5) $\lim_{x\to 0} \frac{\sqrt{x^3+13}\sin(x)}{x}$

(1.6) $\lim_{x\to +\infty} \frac{1}{x-4}$

(1.7) $\lim_{x\to +\infty} \frac{\cos(2x)}{x}$

(1.8) $\lim_{x\to 0} \frac{\sin(x)}{x}$

(1.9) $\lim_{x\to +\infty} e^x \cos(x)$

(1.10) $\lim_{x\to 4} \left[\frac{2}{x-4} - \frac{2}{x^2-7x+12}\right]$

(1.11) $\lim_{x\to 0} \frac{\sin(7x)}{\sin(5x)}$

2. (Give a short written response) What does the derivative tell you about a function?

3. Use the definition of the derivative to compute $f'(x)$.

(3.1) $f(x) = \dfrac{3}{x-1}$

(3.2) $f(x) = 5\sqrt{x+2}$

4. For what values of A and B is $f(x)$ continuous?

$$f(x) = \begin{cases} 6x^{-1} & x < -1 \\ Ax + B & -1 \leq x \leq \frac{1}{2} \\ 8x^{-1} & x > \frac{1}{2} \end{cases}$$

5. For functions, what is the relationship between the concepts: derivative, increasing and decreasing?

6. Find all solutions

(6.1) $3e^x + 5 = e^x + 11$

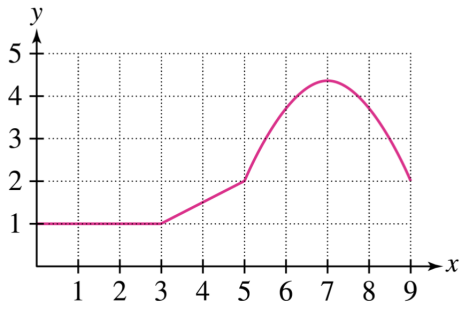
(6.2) $\left(1 + \frac{0.06}{12}\right)^{2x} = 4$

(6.3) $\frac{50}{1 + 2e^{3x}} = 10$

7. Find $f^{-1}(x)$.

(7.1) $f(x) = \frac{1 - 4x}{3x + 2}$

8. Assume the graph of $f(x)$ is below. For what values of a is $f'(a)$ positive, negative and zero?



9. Suppose that f has a domain of [7, 17] and a range of [2, 17].
   (9.1) What are the domain and range of the function y = f(x) + 4?
   (9.2) What are the domain and range of the function y = f(x + 4)?

10. According to Peak Oil Theory, first proposed in 1956, the total amount of crude oil $Q(t)$ produced worldwide up to time t has a graph like the one shown below.

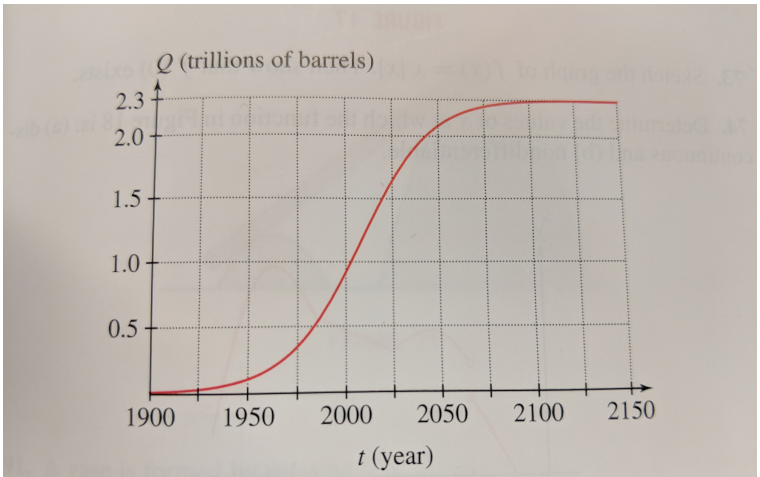


FIGURE 14 Total oil production up to time t.

(10.1) Estimate the average rate of change of oil production from 1900 to 2020. (Hint: This is the slope of the corresponding secant line.)

(10.2) Estimate the instanenous rate of change of oil production at the year 2100. (Hint: This is the derivative at the year 2100.)

(10.3) Compute and interpret $L = \lim_{t\to\infty} Q(t)$.