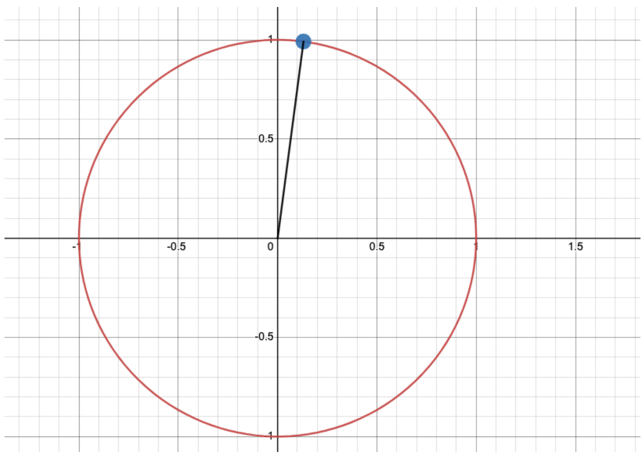
wk 4 — Fri.

Next 3 weeks : Derivatives

Today: derivatives of $f(x) = \sin(x)$, $g(x) = \cos(x)$.



The instantaneous rate of change in y-coord, y =sin(a)

is different, depending what a is.

When a is close to 0, the derivative of sin(a) is big

When a is close to 90 degrees, the derivative of sin(a) is small.

We'll see precisely why, next.

$f(x) = \sin(x)$

$f'(x) =$

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)\cos(h) - 1 \cdot \sin(x) + \sin(h)\cos(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)(\cos h - 1) + \sin(h)\cos(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)(\cos h - 1)}{h} + \frac{\sin(h)\cos(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)(\cos(h) - 1)}{h} + \lim_{h \to 0} \frac{\sin(h)\cos(x)}{h}$$

$$= \sin(x) \cdot \underbrace{\boxed{\lim_{h \to 0} \frac{\cos(h) - 1}{h}}}_{= 0} + \cos(x) \underbrace{\boxed{\lim_{h \to 0} \frac{\sin(h)}{h}}}_{= 1}$$

$$= 0 + \cos(x) \cdot 1 = \cos(x)$$

So $\boxed{\frac{d}{dx}\left(\sin(x)\right) = \cos(x)}$

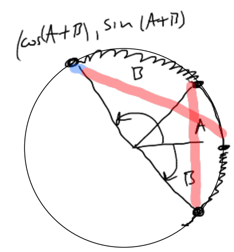
Tools!

def'n: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$ b/c $\cos x - 1 \approx 0$ when ≈ 0

$\sin(A+B) = \sin A \cos B + \sin B \cos A$



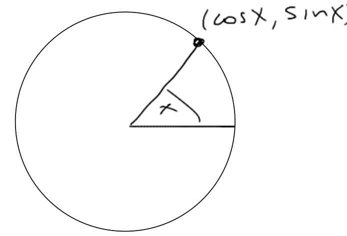
Limit Properties:

limits distribute over sums & multiplication

$$\lim_{h \to 0} 5 \cdot \cos(h) = 5 \lim_{h \to 0} \cos(h)$$

Ex.

what is the derivative of $f(x) = \sin(x)$ when $x = 0$, $x = \pi/2$, $x = -\pi$



Ans $f(x) = \sin x$

$f'(x) = \cos x$

so

$$\left.\begin{array}{l} f'(0) = \cos(0) = 1 \\ f'(\pi/2) = \cos(\pi/2) = 0 \\ f'(-\pi) = \cos(-\pi) = -1 \end{array}\right\}$$

Exercise: Reconcile these answers with:

https://www.desmos.com/calculator/yqwb4hqatl

By a similar calculation we find

$$\frac{d}{dx}(\cos x) = -\sin(x)$$

Exercise

$$\frac{1}{x} = x^{-1}$$

$$f(x) = \underbrace{\frac{1}{x} + x^2 + 3x^4}_{\text{use power rule}} + 5\cos x$$

$$f'(x) = -x^{-2} + 2x + 12x^3 - 5\sin x$$

Facts

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$$

careful not $*$ or $\div$!!!!

$$\frac{d}{dx}\left(k f(x)\right) = k\frac{d}{dx}\left(f(x)\right)$$