Wk 4 —— Mon

If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet after t seconds is given by $y = 40t - 16t^2$.

(a) Find the average velocity for the time period beginning with $t = 2$ and

(1) lasting 0.5 seconds: ____ ft/s —— $[2, 2.5]$ interval $\Rightarrow$ $\frac{y(2.5) - y(2)}{2.5 - 2} = \frac{(40(2.5) - 16(2.5)^2) - (40(2) - 16(2)^2)}{.5}$

(2) lasting 0.1 seconds: ____ ft/s

(3) lasting 0.05 seconds: ____ ft/s

(4) lasting 0.01 seconds: ____ ft/s $[2, 2.01]$

(b) Find the instantaneous velocity when $t = 2$: -24 ft/s $y'(2) = 40 - 32(2) = -24$

derivative of y when $t = 2$

we'll use the Power Rule for derivatives.

$y = 40t - 16t^2$

$y' = 40 - 32t$

now, " t " is the variable, not x so we'll take the derivative with respect to t

$\frac{d}{dt}(y) = y' = \frac{dy}{dt}$

(b) Guess the slope of the tangent line to the curve at P.
(c) Using the slope from part (b), find the equation of the tangent line to the curve at P.

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(b) .5

(c)

$$y - y_1 = m(x - x_1)$$

$$y_1 \to \ln 2 \qquad m \to .5 \qquad x_1 \to 2$$

$$y - \ln 2 = .5(x - 2)$$

$$y = .5x - 1 + \ln 2$$

Power Rule

$f(x) = x^n$, think $n = 2, 3, 4, 5, \ldots$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + nx^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \binom{n}{3}x^{n-3}h^3 + \ldots + \binom{n}{n-1}x^1h^{n-1} + h^n \cancel{- x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(nx^{n-1} + \binom{n}{2}x^{n-2}h^2 + \ldots + h^{n-1}\right)}{\cancel{h}}$$

$$= \lim_{h \to 0} nx^{n-1} + \binom{n}{2}x^{n-2}h^2 + \ldots + h^{n-1} \quad = nx^{n-1}$$

<u>So</u>:

$f(x) = x^3$

$f'(x) = 3x^2$

Patterns —

$(x+h)^2 = x^2 + 2xh + h^2$

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$\downarrow$

$\binom{6}{1}$ "6 chose 1"

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Reminder of two important deriv. properties

- $\frac{d}{dx}$ = symbol for "the derivative $\underbrace{\text{with respect to } x}_{x \text{ is the variable}}$"

① $\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx}\left(f(x)\right) + \frac{d}{dx}\left(g(x)\right)$ piece by piece (works for + / −)

Ex let $F(x) = x^2 + x^3$

$F'(x) = 2x + 3x^2$

② $\frac{d}{dx}\left(k \cdot f(x)\right) = k \cdot \frac{d}{dx}\left(f(x)\right)$ $k \in \mathbb{R}$

Ex let $F(x) = 23x^3$

$F'(x) = 23 \cdot 3x^2 = 69x^2$

Power Rule

works for <u>any</u> $n \in \mathbb{R}$

eg., $n = 1, 2, 3, \ldots$

$n = 1/2, 3/2$

$n = \pi, e,$

<u>Ex</u> $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2} x^{-1/2}$

$= x^{1/2}$

<u>Ex</u> $f(x) = x^{\pi}$, $f'(x) = \pi x^{\pi - 1}$

<u>Ex</u> Careful! the x <u>must</u> be in the <u>base</u>.

$f(x) = 2^{x}$ $f'(x) =$ power rule doesn't apply

$$\boxed{f'(x) \neq x \cdot 2^{x-1}}$$