Wk 4 – Mon

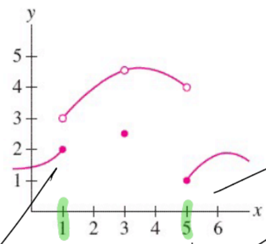
This week: Exam = Thurs

Today: HW Q's + 2.7

limit from right exists & hole filled

2.4.8

Consider the graph of a function.




left cts

right cts

hole filled in

limit exists and it equals f(c) @ x → c

not cts

Find the point *c* at which the function has a $\underbrace{\text{jump discontinuity}}_{\text{limit DNE}}$ but is right-cts.

(Give your answer as an exact number.)

(limit, left + right are finite & not equal)

2.3.5

Evaluate the limit.

$$\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{11}{\underbrace{x^2+5x-24}_{\text{factor }(x-3)(x+8)}}\right) = \frac{1}{0} - \frac{11}{0} = \infty - \infty \Rightarrow \underline{\text{re-write}}$$

$$\underbrace{\lim_{x\to 3}\frac{1}{x-3}}_{DNE} - \lim_{x\to 3}\frac{11}{x^2+5x-24}$$

$\neq$

$$\lim_{x\to 3}\frac{1}{x-3}\cdot\frac{x+8}{x+8} - \frac{11}{(x-3)(x+8)} = \lim_{x\to 3}\frac{x+8-11}{(x-3)(x+8)} = \lim_{x\to 3}\frac{1}{x+8} = \frac{1}{11}$$

direct sub

2.4.7

Define $f(x) = \tan^{-1}\left(\frac{9}{x-1}\right)$ for $x \neq 1$.

Answer the questions using a plot if necessary.

Can $f(1)$ be defined so that $f$ is continuous at $x = 1$?

- ○ Yes
- ☒ No

Need $\lim_{x\to 1} f(x) = f(1)$ # ?

thus this must be some #

How should $f(1)$ be defined so that $f$ is right-continuous at $x = 1$?

(Use symbolic notation and fractions where needed.)

$$\lim_{x\to 1^+} \arctan\left(\frac{9}{x-1}\right) = \tan^{-1}\left(\frac{9}{1.1-1}\right) = \tan^{-1}\left(\frac{9}{.01}\right) \approx \frac{\pi}{2}$$

$$\lim_{x\to 1} \tan^{-1}\left(\frac{9}{x-1}\right) \stackrel{\text{d.s.}}{=} \tan^{-1}\left(\frac{9}{0}\right)$$

$$\frac{9}{0} = \infty$$

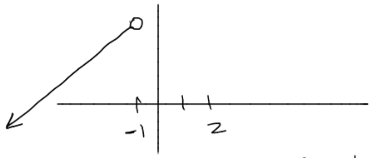
$$= \tan^{-1}(\infty) = \frac{\pi}{2}$$

limit exists if left limit = right limit

$$\lim_{x\to 1^-} \tan^{-1}\left(\frac{9}{x-1}\right) \approx \tan^{-1}\left(\frac{9}{.9-1}\right) \approx \tan\left(\frac{9}{-.01}\right) \approx \tan(-\infty) \approx -\frac{\pi}{2}$$

2.4.10

Let $f$ be the function




$$f(x) = \begin{cases} x + 8 & \text{for } x < -1 \\ cx & \text{for } -1 \le x \le 2 \\ x + 12 & \text{for } x > 2 \end{cases}$$

Find the value of $c$ that makes the function left-continuous.

(Use symbolic notation and fractions where needed.)

Left: look @ left break pt

Need $\underbrace{\lim_{x \to -1^-} f(x)}_{\text{sub } x=-1 \text{ into the } x+8 \text{ branch}} = f(-1)$ sub $x=-1$ into $cx$ branch

$= -1+8 = c(-1)$

left middle

$7 = -c \Rightarrow c = -7$

$c_1 =$

Find the value of $c$ that makes the function right-continuous.

(Use symbolic notation and fractions where needed.)

$\lim_{x \to 2^+} f(x) = f(2)$

$c_2 =$ 7

sub $x=2$ into RIGHT BRANCH

sub $x=2$ into middle

$2+12 = f(2) = 2 \cdot c$

$c=7$

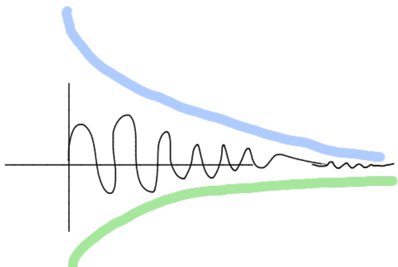
2.7

## Squeeze theorem:

If $h(x) \le f(x) \le g(x)$ for all $x \in \mathbb{R}$

and $\lim_{x \to c} h(x) = L$ $\lim_{x \to c} g(x) = L$

then $\lim_{x \to c} f(x) = L$



Ex $\lim_{x \to \infty} \frac{\sin(x)}{x} =$

① $-1 \le \sin(x) \le 1$

② Since $x \to \infty$, you know $x > 0$ so dividing an inequality by it is legit.

$$\frac{-1}{x} \le \frac{\sin(x)}{x} \le \frac{1}{x}$$

By Squeeze thm conclude: $\lim_{x \to \infty} \frac{\sin(x)}{x} = 0$

③ $\lim_{x \to \infty} \frac{-1}{x} =$ 0 $\lim_{x \to \infty} \frac{1}{x} =$ 0

Limits @ infinity of Rational Functions Theorem

Idea: limit as x -> infinity of a fraction, only depends on the leading terms / coefficients

How: divide top/bottom by the leading term in denom

$$\frac{x}{\sqrt{x^2+100}} \leftarrow \text{"}x\text{"}$$

$$\lim_{x \to \infty} \frac{3x}{x+1000} = \frac{3 \cdot \infty}{\infty + 1000} = \frac{\infty}{\infty}$$

divide top + bottom by x (largest degree term in denom)

take limit of each piece

$$\lim_{x \to \infty} \frac{3x}{x+1000} \cdot \frac{1/x}{1/x} = \lim_{x \to \infty} \frac{3}{1 + \frac{1000}{x}} = \frac{3}{1+0} = 3$$