Wk 4 — Mon

this week:
- Exam 1 = thursday
- Today: • homework Q's
  • 27, 29, 31

3.1.11

Calculate the difference quotient for $f(x) = x + x^{-1}$ at $a = 11$.

① $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(a+h) + (a+h)^{-1} - (a + a^{-1})}{h} = \dfrac{(a+h) + \frac{1}{(a+h)} - a - \frac{1}{a}}{h}$

$= \underbrace{\left[\frac{1}{a+h} - \frac{1}{a} + h\right]}_{\text{common denom}} \frac{1}{h} = \left[\frac{1}{a+h}\left(\frac{a}{a}\right) - \frac{1}{a}\left(\frac{a+h}{a+h}\right) + \frac{h}{1}\left(\frac{a(a+h)}{a(a+h)}\right)\right]\frac{1}{h}$

$= \left[\frac{a - (a+h) + h(a(a+h))}{a(a+h)}\right]\frac{1}{h}$

$= \left[\frac{\overbrace{a - a}^{=0} - h + ha^2 + h^2 a}{a(a+h)}\right]\frac{1}{h} = \frac{h[a^2 + ah - 1]}{a(a+h)} \cdot \frac{1}{h} = \frac{a^2 + ah - 1}{a(a+h)}$

$\dfrac{a^2 + a \cdot 0 - 1}{a(a+0)} =$

direct sub $(h=0)$

② $\lim\limits_{h \to 0} \dfrac{a^2 + ah - 1}{a(a+h)} = \dfrac{a^2 - 1}{a^2} = \boxed{1 - \frac{1}{a^2} = f'(a)}$

the derivative of f at a

③ $f'(11) = 1 - \dfrac{1}{11^2}$

2.7 / More algebraic (algorithmic) ways to compute limits.

the Squeeze Theorem.

If $h(x) \le f(x) \le g(x)$ for all x

and $\lim_{x \to c} h(x) = L$, $\lim_{x \to c} g(x) = L$

then $\boxed{\lim_{x \to c} f(x) = L}$

Ex $\lim_{x \to +\infty} \frac{\sin(x)}{x}$

① know $-1 \le \sin(x) \le 1$ ② $\div$ by x

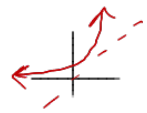
and since $x \to +\infty$, (know $x > 0$) $\Rightarrow$ $-\frac{1}{x} \le \frac{\sin(x)}{x} \le \frac{1}{x}$

③ $\lim_{x \to +\infty} -\frac{1}{x} = 0$ $\lim_{x \to +\infty} \frac{1}{x} = 0$ $\Rightarrow$ $\lim_{x \to +\infty} \frac{\sin x}{x} = 0$

Common Limits to know

$$\lim_{x \to -\infty} x^{\text{even}} = +\infty$$

$$\lim_{x \to -\infty} x^{\text{odd}} = -\infty$$



$$\lim_{x \to -\infty} e^{x} = 0$$

$$\lim_{x \to +\infty} \tan^{-1}(x) = \underbrace{\tan^{-1}(+\infty)}_{\text{arc-tan}} = \frac{\pi}{2}$$

arc-tan = angle that gives $(+\infty)$ slope

$\Rightarrow$ vertical

Limits @ Infinity of Rational Functions Theorem

Idea: · limits @ ∞, of fractions only depend on leading (highest degree term) coefficients. (↳ #'s in front)

· Why?

Ex $\lim_{x\to\infty} \frac{3x+1}{x+100} = \lim_{x\to\infty} \frac{3x}{x+100} + \underbrace{\lim_{x\to\infty}\frac{1}{x+100}}_{=0}$ (~ $\lim_{x\to\infty}\frac{1}{x}$)

$\frac{3x}{x+100} \approx \frac{3x}{x} = 3$

How you'll solve these:

Ex $\lim_{x\to\infty} \frac{3x+1}{x+100} \overset{\text{dis.}}{=} \frac{3\infty+1}{\infty+100} = \frac{\infty}{\infty}$

(try: ÷ top & bottom by "leading degree term in bottom"

$$\lim_{x\to\infty} \frac{3x+1}{x+100} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{3+\frac{1}{x}}{1+\frac{100}{x}} = \frac{3+\overbrace{\lim_{x\to\infty}\frac{1}{x}}^{=0}}{1+\underbrace{\lim_{x\to\infty}\frac{100}{x}}_{=0}} = 3$$

beware: the leading degree term downstairs is different when radicals are involved:

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$$

$\lim_{x\to 5} 3+\frac{10}{x} \overset{\text{dis}}{=} 3+\frac{10}{5}$

" $\lim_{x\to 5} 3 + \lim_{x\to 5}\frac{10}{x}$

" 3

#2

dis. $\frac{\infty}{\sqrt{\infty^2+1}} = \frac{\infty}{\infty}$

÷ by x b/c $\sqrt{x^2+1} \approx \sqrt{x^2} = x$

eg $\sqrt{x^2+3x+7} \approx \sqrt{x^2} = x$

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1}{\frac{\sqrt{x^2+1}}{x}} = \lim_{x\to\infty} \frac{1}{\frac{\sqrt{x^2+1}}{\sqrt{x^2}}} = \lim \frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

(x " $\sqrt{x^2}$)

$$= \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1+0}} = 1$$

$\lim_{x\to\infty} \frac{1}{x^2} = 0$