Math 161 - Calculus - Exam 1 - Guide **Name:** ______________________

September 12, 2024

On the exam you must show your work to receive full credit.

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1. Evaluate the following limits.:

(1.1) $\lim\limits_{x\to 4} \dfrac{1}{x}$

(1.2) $\lim\limits_{x\to 5} 3$

(1.3) $\lim\limits_{x\to 4} \dfrac{1}{x-4}$

$\lim\limits_{x\to 4^-} \dfrac{1}{x-4} = \dfrac{1}{3.9999-4} = \dfrac{1}{-\text{small}} = -\text{Large} = -\infty$ ($x<4$, $= -\infty$)

$\lim\limits_{x\to 4^+} \dfrac{1}{x-4} = +\infty$

$\Rightarrow$ DNE

(1.4) $\lim\limits_{x\to 5} \dfrac{-1}{(x-5)^2} \approx \dfrac{-1}{+\text{small}} = -\text{large} = -\infty$

(1.5) $\lim\limits_{x\to 0} \dfrac{\sqrt{x^3+13}\sin(x)}{x}$

(1.6) $\lim\limits_{x\to +\infty} \dfrac{1}{x-4}$

(1.7) $\lim\limits_{x\to +\infty} \dfrac{\cos(2x)}{x}$

(1.8) $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x}$

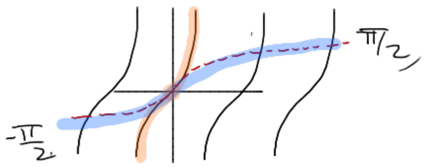
(1.9) $\lim\limits_{x\to +\infty} e^x \cos(x)$

(1.10) $\lim\limits_{x\to 4} \left[\dfrac{2}{x-4} - \dfrac{2}{x^2-7x+12}\right]$

(1.11) $\lim\limits_{x\to 0} \dfrac{\sin(7x)}{\sin(5x)}$

The – w/c 4

$$\lim_{x\to 1} \tan^{-1}\left(\frac{1}{x-1}\right) = \begin{cases} \lim_{x\to 1^-} = -\frac{\pi}{2} \\ \lim_{x\to 1^+} = \frac{\pi}{2} \end{cases}$$



Today: 2.9 (Formal Def'n of Limits)

(height)

Give a formal definition of the limit $\lim_{x \to \infty} f(x) = L$.

M = large # (far to right on # line)

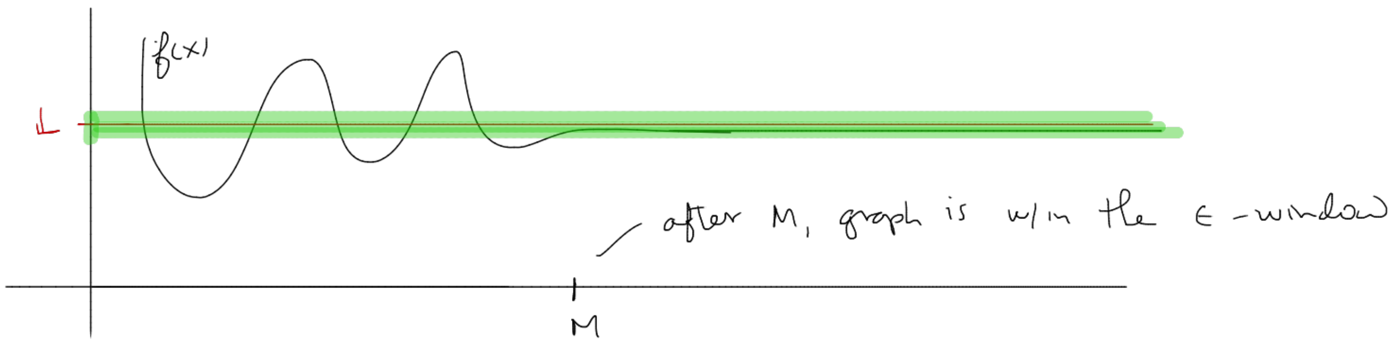
- ◯ $\lim_{x \to \infty} f(x) = L$ if, for any $\epsilon > 0$, there exists an $M > 0$ such that $|f(x) - L| > \epsilon$ whenever $x < M$
- ◯ $\lim_{x \to \infty} f(x) = L$ if, for any $\epsilon > 0$, there exists an $M > 0$ such that $|f(x) - L| < \epsilon$ whenever $x < M$

— graph w/in ϵ-window

- ◯ $\lim_{x \to \infty} f(x) = L$ if, for any $\epsilon > 0$, there exists an $M > 0$ such that $|f(x) - L| < \epsilon$ whenever $x > M$
- ◯ $\lim_{x \to \infty} f(x) = L$ if, for any $\epsilon > 0$, there exists an $M > 0$ such that $|f(x) - L| > \epsilon$ whenever $x > M$

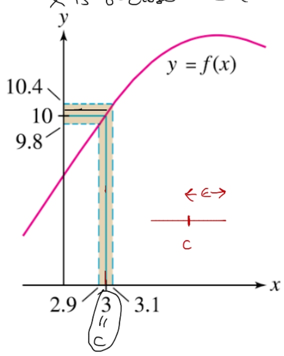
epsilon ⟷ target window (y-axis)

Height



Def'n: $\lim_{x \to c} f(x) = L$ means

If $\underbrace{|x - c| < \delta}_{x \text{ is } \delta\text{-close to } c \text{ (or } x \text{ is within } \delta \text{ distance of } c)}$, then $\overbrace{|f(x) - L|}^{f(x) \text{ is within } \epsilon \text{ of } L} < \epsilon$.

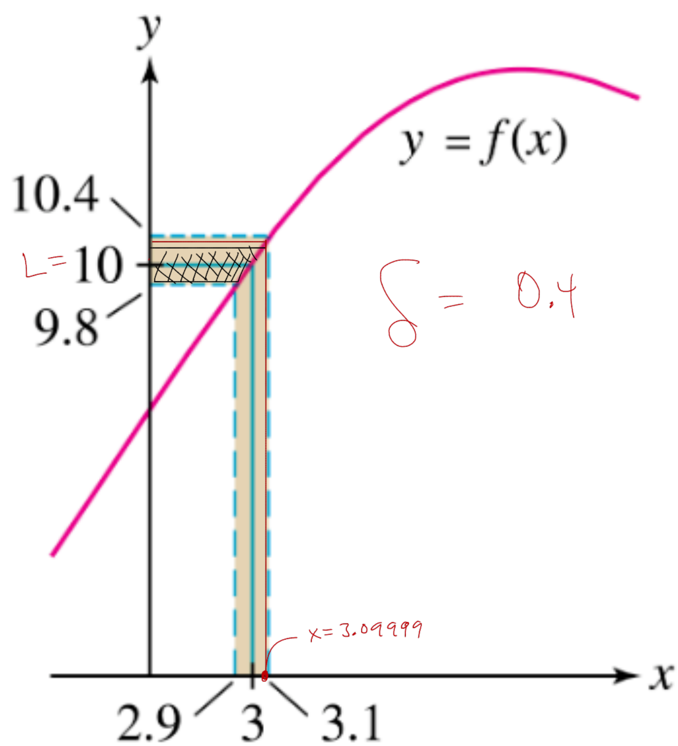


$c = 3$

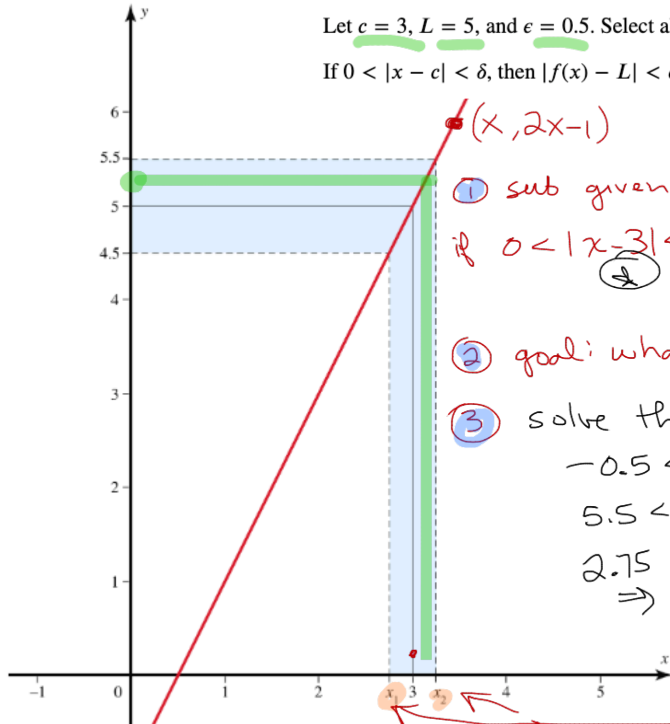
$\delta = 0.4$

$\epsilon = .1$

If $|x - c| < \delta$, then $|f(x) - L| < \epsilon$.



Consider the given graph of the function $f(x) = 2x - 1$.



Let $c = 3$, $L = 5$, and $\epsilon = 0.5$. Select all values of δ such that the given statement holds.

If $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

$(x, 2x-1)$

① sub given info into def

if $0<|x-3|<\delta$ then $|2x-1-5|<0.5$

$|2x-6|<0.5$

② goal: what δ's work?

③ solve this:

$-0.5<2x-6<0.5$

$5.5<2x<6.5$

$2.75<x<3.25$

$\Rightarrow x\in(2.75, 3.25)$

④ So, $\delta = 3.25-3 = 0.25$

That is, plus either 2.75 or 3.25 into ★ get

$0<|2.75-3|<\delta \Rightarrow 0<|-0.25|<\delta$

$0.25<\delta$

Recall: $\lim_{x \to -4} f(x) = 2$ means if $|x-c| < \delta$ then $|f(x)-L| < \epsilon$

w/ $c = -4$, $L = 2$, $f(x) = 2x+10$ $\Big\}$ if $|x+4| < 0.01$ then $|2x+10-2| < \epsilon$ └ unknown (depends on δ)

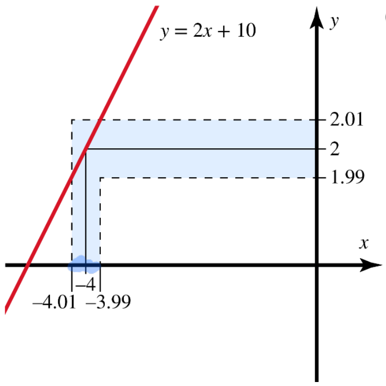
Consider the function $f(x) = 2x + 10$ and the formal definition of the limit, $\lim\limits_{x \to -4} f(x) = 2$.

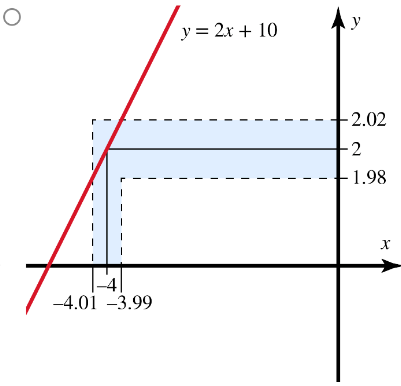
① solve this, take that ans & incorporate into other

Which graph represents $\lim\limits_{x \to -4} f(x) = 2$ for $\delta = 0.01$?

$$-0.01 < x + 4 < 0.01$$

$$-4.01 < x < -3.99$$





② choose one of the endpoints $x \in (-4.01, -3.99)$ & sub into other ineq.

$x = -4.01 \Rightarrow$

$$|2(-4.01) + 8| < \epsilon$$

$$= |-8.02 + 8| = |-0.02| = 0.02$$

So set $\epsilon = 0.02$