Thu: wk 4

Today: - Questions (Review)
- Formal Def'n of Limit

Ex. (Choosing constants to make $f(x)$ cts)

$$f(x) = \begin{cases} 3x^2 & \text{if } x < 1 \\ Ax + B & 1 \le x \le 3 \\ -x + 7 & x > 3 \end{cases} \Rightarrow$$

$A = 3 - 2.5 = 0.5$

use this to find A

Need: $\underbrace{\lim_{x \to 1^-} f(x)}_{\text{sub into } 3x^2} = f(1) = \lim_{x \to 1^+} f(x)$

$3(1)^2 = A(1) + B \Rightarrow 3 = A + B$

combine (solve for A, plug in)

$A = 3 - B$

$\underbrace{\lim_{x \to 3^+} f(x)}_{\text{sub } x=3 \text{ into } -x+7} = f(3) = \lim_{x \to 3^-} f(x)$

$-5 = -2B \Rightarrow B = 2.5$

$-3 + 7 = A \cdot 3 + B \Rightarrow 4 = 3A + B$, so $4 = 3(3 - B) + B) = 9 - 3B + B = 9 - 2B$

(isolate B)

graph w/in window

Give a formal definition of the limit $\lim_{x\to\infty} f(x) = L$.

- ○ $\lim_{x\to\infty} f(x) = L$ if, for any $\epsilon > 0$, there exists an $M > 0$ such that $|f(x) - L| > \epsilon$ whenever $x < M$
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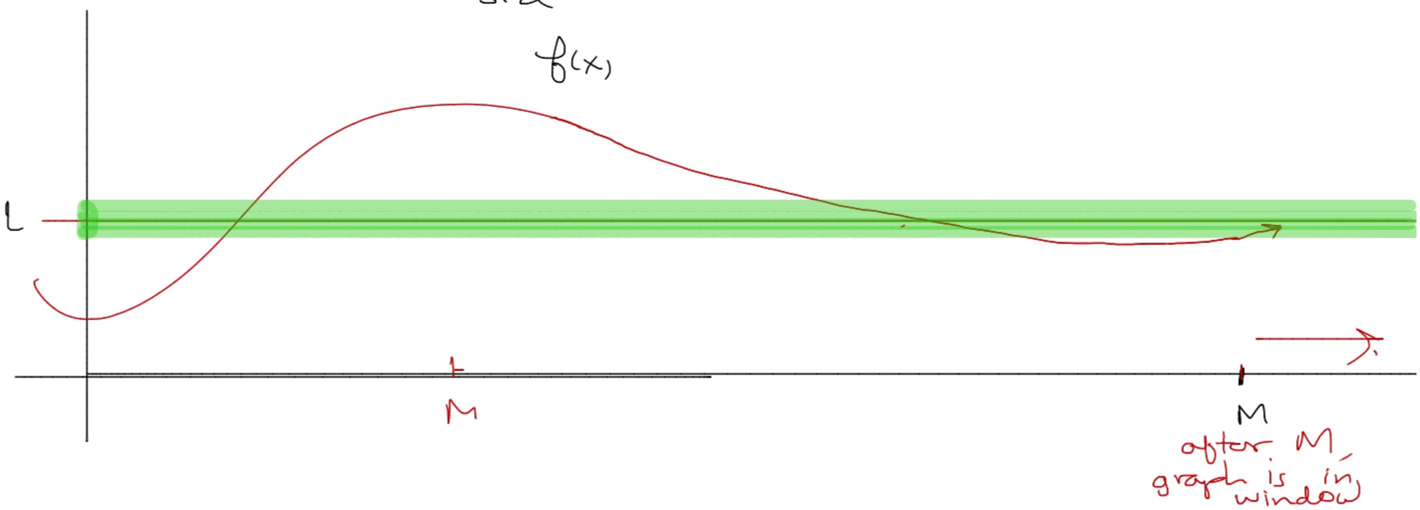
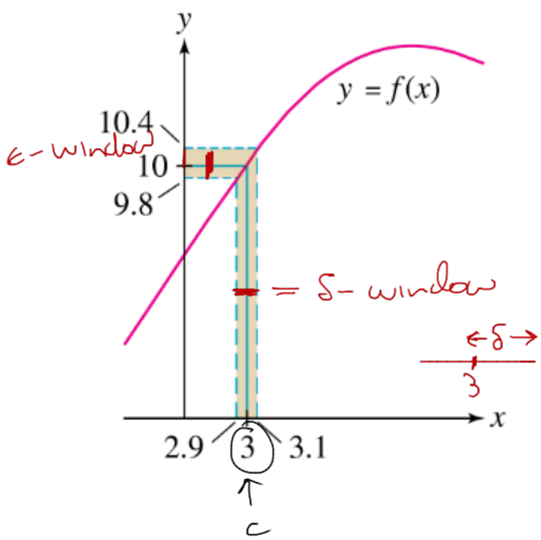
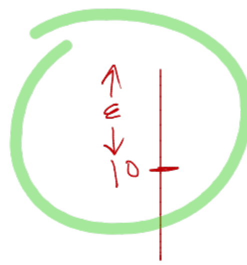
- ○ $\lim_{x\to\infty} f(x) = L$ if, for any $\underbrace{\epsilon}_{\text{window size}} > 0$, there exists an $\underbrace{M}_{\text{far out spot on x-axis}} > 0$ such that $|f(x) - L| > \epsilon$ whenever $x > M$



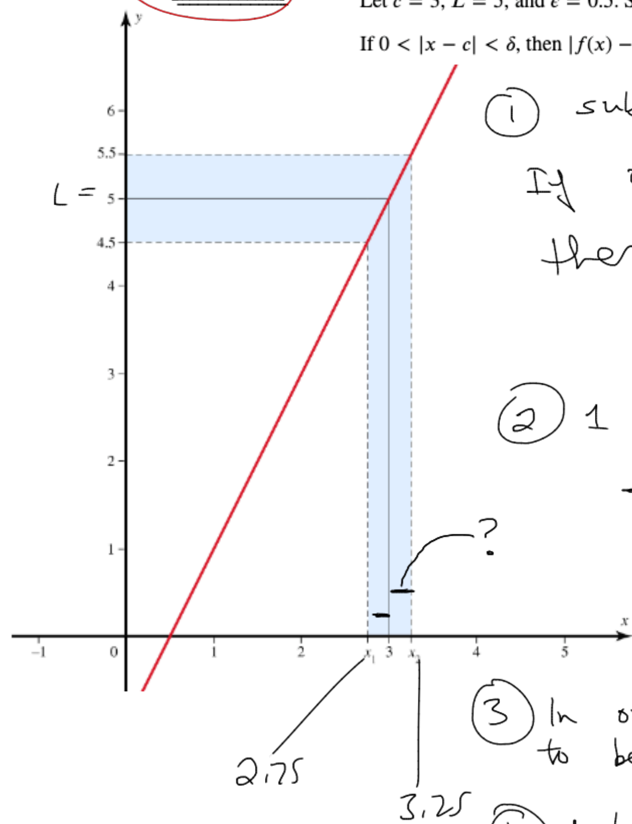
$$\lim_{x \to c} f(x) = L$$

If $|x - c| < \delta$, then $|f(x) - L| < \epsilon$.





Consider the given graph of the function $f(x) = 2x - 1$.



Let $c = 3$, $L = 5$, and $\epsilon = 0.5$. Select all values of δ such that the given statement holds.

If $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

1. sub given info here

If $0 < |x - 3| < \delta$

then

$$|2x - 1 - 5| < 0.5$$

2. 1 unknown here ... solve for x

$$-0.5 < 2x - 6 < 0.5$$

$$5.5 < 2x < 6.5$$

$$2.75 < x < 3.25$$

3. In order for the desired inequality to be true $x \in (2.75, 3.25)$

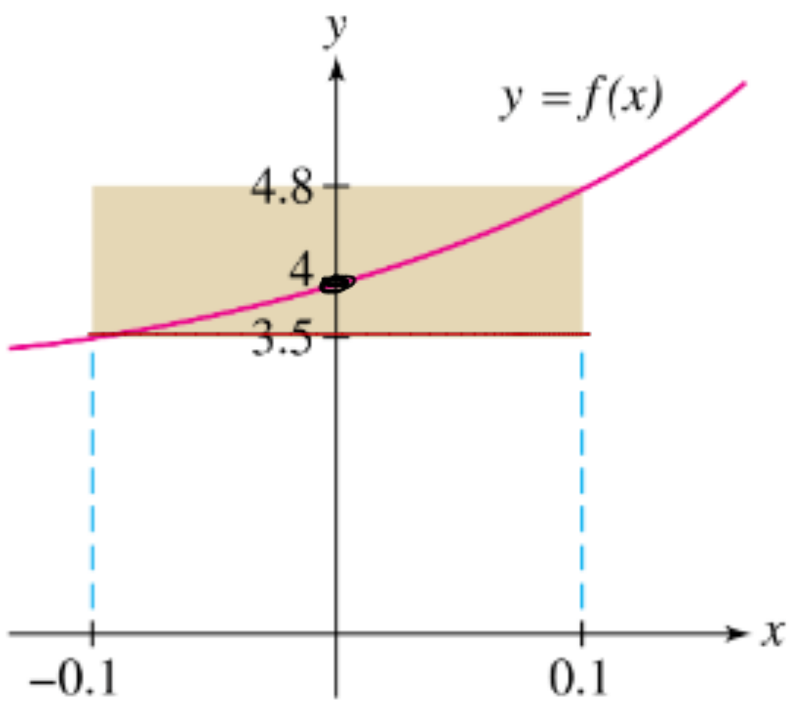
4. sub $x = 2.75$ into δ-inequality

$$0 < |2.75 - 3| < \delta$$

so $.25 < \delta$

5. So $\delta = .25$ is max δ window

If $|x| < \delta$, then $|f(x) - L| < \epsilon$.



$\epsilon = 0.8$

if $|x-c| < \delta$ then $|f(x) - L| < \epsilon$

① sub in given

$|x+4| < 0.01$ then $|2x+10 - 2| < \epsilon$

$|2x+8| < \epsilon$

Consider the function $f(x) = 2x + 10$ and the formal definition of the limit, $\lim_{x \to -4} f(x) = 2$.

② solve $|x+4| < 0.01$

$-0.01 < x+4 < 0.01$

$-4.01 < x < 3.99$

③ sub endpts $x = -4.01$ into other inequal

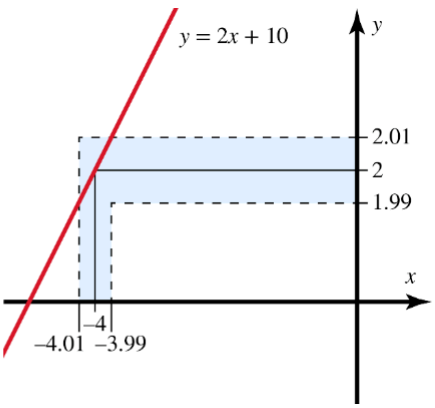
$|2(-4.01) + 8| < \epsilon$

$|-8.02 + 8| < \epsilon$

$|-0.02| < \epsilon$

so $\epsilon = \underline{0.02}$

Which graph represents $\lim_{x \to -4} f(x) = 2$ for $\delta = 0.01$?

- [ ] 

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